Maple 2018.2 Integration Test Results
on the problems in "1 Algebraic functions/1.3 Miscellaneous"

Test results for the 136 problems in "1.3.1 Rational functions.txt"

Problem 1: Result is not expressed in closed-form.

$$\int \frac{1}{-9 b x+9 x^{3}+2 b^{3/2} \sqrt{3}}\, dx$$

Optimal(type 3, 55 leaves, 3 steps):

$$-\frac{\ln\left(-x \sqrt{3}+\sqrt{b}\right)}{27 b}+\frac{\ln\left(x \sqrt{3}+2 \sqrt{b}\right)}{27 b}+\frac{\sqrt{3}}{9 \sqrt{b}\,\left(-3 x+\sqrt{3}\, \sqrt{b}\right)}$$

Result(type 7, 42 leaves):

$$\frac{\left(\sum_{\_R=RootOf\left(-9 b\, \_Z+9 \_Z^{3}+2 b^{3/2} \sqrt{3}\right)} \frac{\ln(x-\_R)}{3 \_R^{2}-b}\right)}{9}$$

Problem 2: Result more than twice size of optimal antiderivative.

$$\int \left(b^{3} x^{3}+3 a b^{2} x^{2}+3 a^{2} b x+a^{3}\right)^{3} dx$$

Optimal(type 1, 12 leaves, 2 steps):

$$\frac{(b x+a)^{10}}{10 b}$$

Result(type 1, 97 leaves):

$$\frac{1}{10} b^{9} x^{10}+a b^{8} x^{9}+\frac{9}{2} a^{2} b^{7} x^{8}+12 a^{3} b^{6} x^{7}+21 a^{4} b^{5} x^{6}+\frac{126}{5} a^{5} b^{4} x^{5}+21 a^{6} b^{3} x^{4}+12 a^{7} b^{2} x^{3}+\frac{9}{2} a^{8} b x^{2}+a^{9} x$$

Problem 5: Result more than twice size of optimal antiderivative.

$$\int \left(a c e+(a c f+a d e+b c e) x+(a d f+b c f+b d e) x^{2}+b d f x^{3}\right)^{3} dx$$

Optimal(type 1, 347 leaves, 3 steps):

$$\frac{(-a d+b c)^{3} (-a f+b e)^{3} (b x+a)^{4}}{4 b^{7}}+\frac{3 (-a d+b c)^{2} (-a f+b e)^{2} (-2 a d f+b c f+b d e) (b x+a)^{5}}{5 b^{7}}$$
$$+\frac{(-a d+b c) (-a f+b e) \left(5 a^{2} d^{2} f^{2}-5 a b d f(c f+d e)+b^{2} \left(c^{2} f^{2}+3 c d e f+d^{2} e^{2}\right)\right) (b x+a)^{6}}{2 b^{7}}$$
$$+\frac{(-2 a d f+b c f+b d e) \left(10 a^{2} d^{2} f^{2}-10 a b d f(c f+d e)+b^{2} \left(c^{2} f^{2}+8 c d e f+d^{2} e^{2}\right)\right) (b x+a)^{7}}{7 b^{7}}$$

$$+\frac{3df\left(5a^2d^2f^2-5abdf(cf+de)+b^2\left(c^2f^2+3cdef+d^2e^2\right)\right)(bx+a)^8}{8b^7}+\frac{d^2f^2(-2adf+bcf+bde)(bx+a)^9}{3b^7}+\frac{d^3f^3(bx+a)^{10}}{10b^7}$$

Result(type 1, 860 leaves):

$$\frac{b^3d^3f^3x^{10}}{10}+\frac{(adf+bcf+bde)b^2d^2f^2x^9}{3}$$
$$+\frac{\left((acf+ade+bce)b^2d^2f^2+2(adf+bcf+bde)^2bdf+bdf\left(2(acf+ade+bce)bdf+(adf+bcf+bde)^2\right)\right)x^8}{8}+\frac{1}{7}\Big(\Big(aceb^2d^2f^2$$
$$+2(acf+ade+bce)(adf+bcf+bde)bdf+(adf+bcf+bde)\left(2(acf+ade+bce)bdf+(adf+bcf+bde)^2\right)+bdf(2acebdf$$
$$+2(acf+ade+bce)(adf+bcf+bde))\Big)x^7\Big)+\frac{1}{6}\Big(\Big(2ace(adf+bcf+bde)bdf+(acf+ade+bce)\big(2(acf+ade+bce)bdf$$
$$+(adf+bcf+bde)^2\big)+(adf+bcf+bde)(2acebdf+2(acf+ade+bce)(adf+bcf+bde))+bdf\big(2ace(adf+bcf+bde)$$
$$+(acf+ade+bce)^2\big)\Big)x^6\Big)+\frac{1}{5}\Big(\Big(ace\left(2(acf+ade+bce)bdf+(adf+bcf+bde)^2\right)+(acf+ade+bce)(2acebdf+2(acf$$
$$+ade+bce)(adf+bcf+bde))+(adf+bcf+bde)\left(2ace(adf+bcf+bde)+(acf+ade+bce)^2\right)+2bdface(acf+ade+bce)\Big)x^5\Big)$$
$$+\frac{1}{4}\Big(\Big(ace(2acebdf+2(acf+ade+bce)(adf+bcf+bde))+(acf+ade+bce)\left(2ace(adf+bcf+bde)+(acf+ade+bce)^2\right)$$
$$+2(adf+bcf+bde)ace(acf+ade+bce)+bdfa^2c^2e^2\Big)x^4\Big)$$
$$+\frac{\left(ace\left(2ace(adf+bcf+bde)+(acf+ade+bce)^2\right)+2(acf+ade+bce)^2ace+(adf+bcf+bde)a^2c^2e^2\right)x^3}{3}$$
$$+\frac{3a^2c^2e^2(acf+ade+bce)x^2}{2}+a^3c^3e^3x$$

Problem 11: Unable to integrate problem.

$$\int\left(dx^3+cx^2\right)^n\,dx$$

Optimal(type 5, 57 leaves, 3 steps):

$$\frac{x\left(dx^3+cx^2\right)^n\operatorname{hypergeom}\left([-n,1+2n],[2+2n],-\frac{dx}{c}\right)}{(1+2n)\left(1+\frac{dx}{c}\right)^n}$$

Result(type 8, 15 leaves):

$$\int\left(dx^3+cx^2\right)^n\,dx$$

Problem 17: Result is not expressed in closed-form.

$$\int\frac{1}{8x^4-x^3+8x+8}\,dx$$

Optimal(type 3, 192 leaves, 16 steps):

$$-\frac{\arctan\left(\frac{\left(3-\left(1+\frac{4}{x}\right)^2\right)\sqrt{7}}{42}\right)\sqrt{7}}{84}-\frac{\ln\left(\left(1+\frac{4}{x}\right)^2+3\sqrt{29}-\left(1+\frac{4}{x}\right)\sqrt{6+6\sqrt{29}}\right)\sqrt{-132762+81606\sqrt{29}}}{29232}$$

$$+\frac{\ln\left(\left(1+\frac{4}{x}\right)^2+3\sqrt{29}+\left(1+\frac{4}{x}\right)\sqrt{6+6\sqrt{29}}\right)\sqrt{-132762+81606\sqrt{29}}}{29232}-\frac{\arctan\left(\frac{2+\frac{8}{x}-\sqrt{6+6\sqrt{29}}}{\sqrt{-6+6\sqrt{29}}}\right)\sqrt{132762+81606\sqrt{29}}}{14616}$$

$$-\frac{\arctan\left(\frac{2+\frac{8}{x}+\sqrt{6+6\sqrt{29}}}{\sqrt{-6+6\sqrt{29}}}\right)\sqrt{132762+81606\sqrt{29}}}{14616}$$

Result(type 7, 40 leaves):

$$\sum_{\_R=RootOf(8\_Z^4-\_Z^3+8\_Z+8)}\frac{\ln(x-\_R)}{32\_R^3-3\_R^2+8}$$

Problem 18: Result is not expressed in closed-form.

$$\int\frac{1}{\left(4x^4+4x^2+4x+1\right)^2}\,dx$$

Optimal(type 3, 239 leaves, 17 steps):

$$\frac{-17+\left(1+\frac{1}{x}\right)^2}{2\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)}+\frac{\left(59-17\left(1+\frac{1}{x}\right)^2\right)\left(1+\frac{1}{x}\right)}{10\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)}+\frac{7\arctan\left(-\frac{1}{2}+\frac{\left(1+\frac{1}{x}\right)^2}{2}\right)}{4}$$

$$+\frac{\ln\left(\left(1+\frac{1}{x}\right)^2+\sqrt{5}-\left(1+\frac{1}{x}\right)\sqrt{2+2\sqrt{5}}\right)\sqrt{-59590+26650\sqrt{5}}}{400}-\frac{\ln\left(\left(1+\frac{1}{x}\right)^2+\sqrt{5}+\left(1+\frac{1}{x}\right)\sqrt{2+2\sqrt{5}}\right)\sqrt{-59590+26650\sqrt{5}}}{400}$$

$$-\frac{\arctan\left(\frac{2+\frac{2}{x}-\sqrt{2+2\sqrt{5}}}{\sqrt{-2+2\sqrt{5}}}\right)\sqrt{59590+26650\sqrt{5}}}{200}-\frac{\arctan\left(\frac{2+\frac{2}{x}+\sqrt{2+2\sqrt{5}}}{\sqrt{-2+2\sqrt{5}}}\right)\sqrt{59590+26650\sqrt{5}}}{200}$$

Result(type 7, 78 leaves):

$$\frac{\frac{9}{20}x^3-\frac{1}{5}x^2+\frac{21}{40}x+\frac{19}{80}}{x^4+x^2+x+\frac{1}{4}}+\frac{\left(\sum_{\_R=RootOf(4\_Z^4+4\_Z^2+4\_Z+1)}\frac{(18\_R^2-16\_R+27)\ln(x-\_R)}{4\_R^3+2\_R+1}\right)}{40}$$

Problem 20: Result more than twice size of optimal antiderivative.

$$\int(b^5x^5+5ab^4x^4+10a^2b^3x^3+10a^3b^2x^2+5a^4bx+a^5)^2\,dx$$

Optimal(type 1, 12 leaves, 2 steps):

$$\frac{(bx+a)^{11}}{11b}$$

Result(type 1, 108 leaves):

$$\frac{1}{11}b^{10}x^{11}+ab^9x^{10}+5a^2b^8x^9+15a^3b^7x^8+30a^4b^6x^7+42a^5b^5x^6+42a^6b^4x^5+30a^7b^3x^4+15a^8b^2x^3+5a^9bx^2+a^{10}x$$

Problem 21: Result more than twice size of optimal antiderivative.

$$\int(b^5x^5+5ab^4x^4+10a^2b^3x^3+10a^3b^2x^2+5a^4bx+a^5)\,dx$$

Optimal(type 1, 12 leaves, 1 step):

$$\frac{(bx+a)^6}{6b}$$

Result(type 1, 53 leaves):

$$a^5x+\frac{5}{2}a^4bx^2+\frac{10}{3}a^3b^2x^3+\frac{5}{2}a^2b^3x^4+ab^4x^5+\frac{1}{6}b^5x^6$$

Problem 27: Result more than twice size of optimal antiderivative.

$$\int\frac{1}{1-(dx+c)^2}\,dx$$

Optimal(type 3, 10 leaves, 2 steps):

$$\frac{\operatorname{arctanh}(dx+c)}{d}$$

Result(type 3, 25 leaves):

$$\frac{\ln(dx+c+1)}{2d}-\frac{\ln(dx+c-1)}{2d}$$

Problem 29: Result more than twice size of optimal antiderivative.

$$\int \frac{x^2}{\sqrt{1-(bx+a)^2}}\,dx$$

Optimal(type 3, 57 leaves, 4 steps):

$$\frac{(2a^2+1)\arcsin(bx+a)}{2b^3}+\frac{3a\sqrt{1-(bx+a)^2}}{2b^3}-\frac{x\sqrt{1-(bx+a)^2}}{2b^2}$$

Result(type 3, 151 leaves):

$$-\frac{x\sqrt{-b^2x^2-2abx-a^2+1}}{2b^2}+\frac{3a\sqrt{-b^2x^2-2abx-a^2+1}}{2b^3}+\frac{a^2\arctan\left(\frac{\sqrt{b^2}\left(x+\frac{a}{b}\right)}{\sqrt{-b^2x^2-2abx-a^2+1}}\right)}{b^2\sqrt{b^2}}+\frac{\arctan\left(\frac{\sqrt{b^2}\left(x+\frac{a}{b}\right)}{\sqrt{-b^2x^2-2abx-a^2+1}}\right)}{2b^2\sqrt{b^2}}$$

Problem 30: Result more than twice size of optimal antiderivative.

$$\int \frac{x^2}{\sqrt{1+(bx+a)^2}}\,dx$$

Optimal(type 3, 53 leaves, 4 steps):

$$-\frac{(-2a^2+1)\operatorname{arcsinh}(bx+a)}{2b^3}-\frac{3a\sqrt{1+(bx+a)^2}}{2b^3}+\frac{x\sqrt{1+(bx+a)^2}}{2b^2}$$

Result(type 3, 145 leaves):

$$\frac{x\sqrt{b^2x^2+2abx+a^2+1}}{2b^2}-\frac{3a\sqrt{b^2x^2+2abx+a^2+1}}{2b^3}+\frac{a^2\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{b^2\sqrt{b^2}}$$
$$-\frac{\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{2b^2\sqrt{b^2}}$$

Problem 31: Result is not expressed in closed-form.

$$\int \frac{1}{a+b(dx+c)^4}\,dx$$

Optimal(type 3, 156 leaves, 10 steps):

$$\frac{\arctan\left(-1+\frac{b^{1/4}(dx+c)\sqrt{2}}{a^{1/4}}\right)\sqrt{2}}{4a^{3/4}b^{1/4}d}+\frac{\arctan\left(1+\frac{b^{1/4}(dx+c)\sqrt{2}}{a^{1/4}}\right)\sqrt{2}}{4a^{3/4}b^{1/4}d}-\frac{\ln\left(-a^{1/4}b^{1/4}(dx+c)\sqrt{2}+\sqrt{a}+(dx+c)^2\sqrt{b}\right)\sqrt{2}}{8a^{3/4}b^{1/4}d}$$

$$+\frac{\ln\left(a^{1/4} b^{1/4} (d x+c)\sqrt{2}+\sqrt{a}+(d x+c)^2\sqrt{b}\right)\sqrt{2}}{8 a^{3/4} b^{1/4} d}$$

Result(type 7, 93 leaves):

$$\frac{\sum_{\_R=RootOf(b d^4 \_Z^4+4 d^3 c b \_Z^3+6 c^2 d^2 b \_Z^2+4 c^3 d b \_Z+b c^4+a)} \frac{\ln(x-\_R)}{d^3 \_R^3+3 d^2 c \_R^2+3 c^2 d \_R+c^3}}{4 b d}$$

Problem 35: Result is not expressed in closed-form.

$$\int \frac{1}{-x^4+4 x^3-8 x^2+a+8 x}\,dx$$

Optimal(type 3, 65 leaves, 4 steps):

$$-\frac{\arctan\left(\frac{-1+x}{\sqrt{1-\sqrt{4+a}}}\right)}{2\sqrt{4+a}\sqrt{1-\sqrt{4+a}}}+\frac{\arctan\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right)}{2\sqrt{4+a}\sqrt{1+\sqrt{4+a}}}$$

Result(type 7, 48 leaves):

$$-\frac{\left(\sum_{\_R=RootOf(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a)} \frac{\ln(x-\_R)}{\_R^3-3\_R^2+4\_R-2}\right)}{4}$$

Problem 36: Result is not expressed in closed-form.

$$\int \frac{1}{(-x^4+4 x^3-8 x^2+a+8 x)^3}\,dx$$

Optimal(type 3, 219 leaves, 6 steps):

$$\frac{(5+a+(-1+x)^2)(-1+x)}{8(a^2+7a+12)(3+a-2(-1+x)^2-(-1+x)^4)^2}+\frac{((6+a)(25+7a)+6(7+2a)(-1+x)^2)(-1+x)}{32(3+a)^2(4+a)^2(3+a-2(-1+x)^2-(-1+x)^4)}$$
$$-\frac{3\arctan\left(\frac{-1+x}{\sqrt{1-\sqrt{4+a}}}\right)\left(80+7a^2+14\sqrt{4+a}+a\left(47+4\sqrt{4+a}\right)\right)}{64(3+a)^2(4+a)^{5/2}\sqrt{1-\sqrt{4+a}}}-\frac{3\arctan\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right)\left(14+4a+\frac{-7a^2-47a-80}{\sqrt{4+a}}\right)}{64(3+a)^2(4+a)^2\sqrt{1+\sqrt{4+a}}}$$

Result(type 7, 397 leaves):

$$-\frac{1}{(x^4-4x^3+8x^2-a-8x)^2}\left(\frac{3(7+2a)x^7}{16(a^4+14a^3+73a^2+168a+144)}-\frac{21(7+2a)x^6}{16(a^2+8a+16)(a^2+6a+9)}+\frac{(7a^2+343a+1116)x^5}{32(a^4+14a^3+73a^2+168a+144)}\right.$$
$$-\frac{5(7a^2+175a+528)x^4}{32(a^4+14a^3+73a^2+168a+144)}+\frac{(34a^2+679a+1968)x^3}{16(a^4+14a^3+73a^2+168a+144)}-\frac{(32a^2+623a+1800)x^2}{16(a^4+14a^3+73a^2+168a+144)}$$

$$-\frac{(11a^3+107a^2-84a-1152)x}{32(a^4+14a^3+73a^2+168a+144)}+\frac{11a^3+131a^2+408a+288}{32(a^4+14a^3+73a^2+168a+144)}\Bigg)$$

$$-\frac{3\left(\sum_{\_R=RootOf(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a)}\frac{(108+2(7+2a)\_R^2+4(-2a-7)\_R+7a^2+55a)\ln(x-\_R)}{(\_R^3-3\_R^2+4\_R-2)(a^3+10a^2+33a+36)(4+a)}\right)}{128}$$

Problem 41: Result is not expressed in closed-form.

$$\int\frac{1}{x^2(b^3x^6+9ab^2x^4+27a^2cx^3+27a^2bx^2+27a^3)}\,dx$$

Optimal(type 3, 456 leaves, 14 steps):

$$-\frac{1}{27a^3x}-\frac{(2b-3a^{1/3}c^{2/3})\ln(3a+3a^{2/3}c^{1/3}x+bx^2)}{486a^{11/3}c^{1/3}}+\frac{(2b-3(-1)^{2/3}a^{1/3}c^{2/3})\ln(3a-3(-1)^{1/3}a^{2/3}c^{1/3}x+bx^2)}{162(1+(-1)^{1/3})^2a^{11/3}c^{1/3}}$$

$$+\frac{(-1)^{1/3}(2b+3(-1)^{1/3}a^{1/3}c^{2/3})\ln(3a+3(-1)^{2/3}a^{2/3}c^{1/3}x+bx^2)}{486a^{11/3}c^{1/3}}$$

$$+\frac{(2b^2-12a^{1/3}bc^{2/3}+9a^{2/3}c^{4/3})\arctan\left(\frac{(3a^{2/3}c^{1/3}+2bx)\sqrt{3}}{3\sqrt{a}\sqrt{4b-3a^{1/3}c^{2/3}}}\right)\sqrt{3}}{729a^{23/6}c^{2/3}\sqrt{4b-3a^{1/3}c^{2/3}}}$$

$$+\frac{(-1)^{2/3}(2b^2+12(-1)^{1/3}a^{1/3}bc^{2/3}+9(-1)^{2/3}a^{2/3}c^{4/3})\arctan\left(\frac{(3(-1)^{2/3}a^{2/3}c^{1/3}+2bx)\sqrt{3}}{3\sqrt{a}\sqrt{4b+3(-1)^{1/3}a^{1/3}c^{2/3}}}\right)\sqrt{3}}{243(1-(-1)^{1/3})(1+(-1)^{1/3})^2a^{23/6}c^{2/3}\sqrt{4b+3(-1)^{1/3}a^{1/3}c^{2/3}}}$$

$$+\frac{(2(-1)^{2/3}b^2+12(-1)^{1/3}a^{1/3}bc^{2/3}+9a^{2/3}c^{4/3})\arctan\left(\frac{(3(-1)^{1/3}a^{2/3}c^{1/3}-2bx)\sqrt{3}}{3\sqrt{a}\sqrt{4b-3(-1)^{2/3}a^{1/3}c^{2/3}}}\right)\sqrt{3}}{243(1+(-1)^{1/3})^2a^{23/6}c^{2/3}\sqrt{4b-3(-1)^{2/3}a^{1/3}c^{2/3}}}$$

Result(type 7, 132 leaves):

$$\frac{\sum_{\_R=RootOf(b^3\_Z^6+9ab^2\_Z^4+27ca^2\_Z^3+27a^2b\_Z^2+27a^3)}\frac{(-\_R^4b^3-9\_R^2ab^2-27\_Ra^2c-27a^2b)\ln(x-\_R)}{2\_R^5b^3+12\_R^3ab^2+27\_R^2a^2c+18\_Ra^2b}}{81a^3}-\frac{1}{27a^3x}$$

Problem 42: Result is not expressed in closed-form.

$$\int\frac{x}{x^6+18x^4+324x^3+108x^2+216}\,dx$$

Optimal(type 3, 242 leaves, 14 steps):

$$\frac{(-1)^{2/3}\ln\left(6-3(-3)^{1/3}2^{2/3}x+x^2\right)2^{2/3}3^{1/3}}{1296\left(1+(-1)^{1/3}\right)^2}-\frac{(-1)^{2/3}\ln\left(6+3(-2)^{2/3}3^{1/3}x+x^2\right)2^{2/3}3^{1/3}}{3888}-\frac{\ln\left(6+3\,2^{2/3}3^{1/3}x+x^2\right)2^{2/3}3^{1/3}}{3888}$$
$$-\frac{\arctan\left(\frac{3(-3)^{1/3}2^{2/3}-2x}{\sqrt{24-18(-3)^{2/3}2^{1/3}}}\right)2^{5/6}3^{1/6}}{216\left(1+(-1)^{1/3}\right)^2\sqrt{4-3(-3)^{2/3}2^{1/3}}}+\frac{\operatorname{arctanh}\left(\frac{2^{1/6}\left(3\,3^{1/3}+2^{1/3}x\right)}{\sqrt{-12+9\,2^{1/3}3^{2/3}}}\right)2^{5/6}3^{1/6}}{648\sqrt{-4+3\,2^{1/3}3^{2/3}}}$$
$$+\frac{(-1)^{1/3}\arctan\left(\frac{3(-2)^{2/3}3^{1/3}+2x}{\sqrt{24+18(-2)^{1/3}3^{2/3}}}\right)2^{1/3}3^{1/6}}{324\sqrt{8+9I\,2^{1/3}3^{1/6}+3\,2^{1/3}3^{2/3}}}$$

Result(type 7, 53 leaves):

$$\frac{\left(\sum_{\_R=RootOf(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216)}\frac{\_R\ln(x-\_R)}{\_R^5+12\_R^3+162\_R^2+36\_R}\right)}{6}$$

Problem 43: Result is not expressed in closed-form.

$$\int\frac{1}{x^6+18x^4+324x^3+108x^2+216}\,dx$$

Optimal(type 3, 255 leaves, 14 steps):

$$-\frac{\ln\left(6-3(-3)^{1/3}2^{2/3}x+x^2\right)2^{1/3}3^{2/3}}{1296\left(1+(-1)^{1/3}\right)^2}-\frac{(-1)^{1/3}3^{2/3}\ln\left(6+3(-2)^{2/3}3^{1/3}x+x^2\right)2^{1/3}}{3888}+\frac{\ln\left(6+3\,2^{2/3}3^{1/3}x+x^2\right)2^{1/3}3^{2/3}}{3888}$$
$$+\frac{(-1)^{2/3}\left(3(-3)^{2/3}-2^{2/3}\right)\arctan\left(\frac{3(-3)^{1/3}2^{2/3}-2x}{\sqrt{24-18(-3)^{2/3}2^{1/3}}}\right)3^{5/6}}{972\left(1+(-1)^{1/3}\right)^2\sqrt{8-6(-3)^{2/3}2^{1/3}}}-\frac{\left(9-2^{2/3}3^{1/3}\right)\operatorname{arctanh}\left(\frac{2^{1/6}\left(3\,3^{1/3}+2^{1/3}x\right)}{\sqrt{-12+9\,2^{1/3}3^{2/3}}}\right)}{972\sqrt{-24+18\,2^{1/3}3^{2/3}}}$$
$$+\frac{\left(9-(-2)^{2/3}3^{1/3}\right)\arctan\left(\frac{3(-2)^{2/3}3^{1/3}+2x}{\sqrt{24+18(-2)^{1/3}3^{2/3}}}\right)}{972\sqrt{24+27I\,2^{1/3}3^{1/6}+9\,2^{1/3}3^{2/3}}}$$

Result(type 7, 52 leaves):

$$\frac{\left(\sum_{\_R=RootOf(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216)}\frac{\ln(x-\_R)}{\_R^5+12\_R^3+162\_R^2+36\_R}\right)}{6}$$

Problem 44: Result is not expressed in closed-form.

$$\int\frac{x^7}{\left(x^6+18x^4+324x^3+108x^2+216\right)^2}\,dx$$

Optimal(type 3, 694 leaves, 23 steps):

$$\frac{\left(-4\left(-1\right)^{1/3}3^{2/3}-18\,6^{1/3}+9\left(\left(-2\right)^{2/3}+2\left(-1\right)^{1/3}3^{2/3}\right)x\right)2^{1/3}}{1944\left(1+\left(-1\right)^{1/3}\right)^{4}\left(4-3\left(-3\right)^{2/3}2^{1/3}\right)\left(6-3\left(-3\right)^{1/3}2^{2/3}x+x^{2}\right)}+\frac{-\left(-6\right)^{1/3}\left(9\left(-2\right)^{1/3}+2\,3^{1/3}\right)+9\left(1+\left(-2\right)^{1/3}3^{2/3}\right)x}{4374\left(8+9\,I\,2^{1/3}3^{1/6}+3\,2^{1/3}3^{2/3}\right)\left(6+3\left(-2\right)^{2/3}3^{1/3}x+x^{2}\right)}$$

$$+\frac{\left(4-6\,2^{1/3}3^{2/3}-3\left(6-2^{2/3}3^{1/3}\right)x\right)2^{1/3}3^{2/3}}{17496\left(4-3\,2^{1/3}3^{2/3}\right)\left(6+3\,2^{2/3}3^{1/3}x+x^{2}\right)}+\frac{I\ln\left(6-3\left(-3\right)^{1/3}2^{2/3}x+x^{2}\right)2^{1/3}3^{1/6}}{3888\left(1+\left(-1\right)^{1/3}\right)^{5}}-\frac{\ln\left(6+3\,2^{2/3}3^{1/3}x+x^{2}\right)2^{1/3}3^{2/3}}{104976}$$

$$-\frac{\left(-1\right)^{1/3}\left(\left(-3\right)^{1/3}+3\,2^{1/3}\right)\arctan\left(\frac{2^{1/6}\left(3\left(-3\right)^{1/3}-2^{1/3}x\right)}{\sqrt{12-9\left(-3\right)^{2/3}2^{1/3}}}\right)3^{1/6}\sqrt{2}}{324\left(1+\left(-1\right)^{1/3}\right)^{4}\left(4-3\left(-3\right)^{2/3}2^{1/3}\right)^{3/2}}-\frac{\ln\left(6+3\left(-2\right)^{2/3}3^{1/3}x+x^{2}\right)\left(I+\sqrt{3}\right)2^{1/3}3^{1/6}}{7776\left(1+\left(-1\right)^{1/3}\right)^{5}}$$

$$+\frac{\left(1+\left(-2\right)^{1/3}3^{2/3}\right)\arctan\left(\frac{3\left(-2\right)^{2/3}3^{1/3}+2x}{\sqrt{24+18\left(-2\right)^{1/3}3^{2/3}}}\right)\sqrt{6}}{324\left(1-\left(-1\right)^{1/3}\right)^{2}\left(1+\left(-1\right)^{1/3}\right)^{4}\left(4+3\left(-2\right)^{1/3}3^{2/3}\right)^{3/2}}+\frac{\left(1-2^{1/3}3^{2/3}\right)\operatorname{arctanh}\left(\frac{2^{1/6}\left(3\,3^{1/3}+2^{1/3}x\right)}{\sqrt{-12+9\,2^{1/3}3^{2/3}}}\right)\sqrt{6}}{324\left(1-\left(-1\right)^{1/3}\right)^{2}\left(1+\left(-1\right)^{1/3}\right)^{4}\left(-4+3\,2^{1/3}3^{2/3}\right)^{3/2}}$$

$$+\frac{\arctan\left(\frac{3\left(-3\right)^{1/3}2^{2/3}-2x}{\sqrt{24-18\left(-3\right)^{2/3}2^{1/3}}}\right)\left(9I+3^{1/3}\left(2I\,2^{2/3}-9\,3^{1/6}+2\,2^{2/3}\sqrt{3}\right)\right)}{5832\left(1+\left(-1\right)^{1/3}\right)^{5}\sqrt{8-6\left(-3\right)^{2/3}2^{1/3}}}$$

$$+\frac{\left(9\,3^{1/6}+I\left(4\,2^{2/3}-3\,3^{2/3}\right)\right)\arctan\left(\frac{3\left(-2\right)^{2/3}3^{1/3}+2x}{\sqrt{24+18\left(-2\right)^{1/3}3^{2/3}}}\right)3^{1/3}}{5832\left(1+\left(-1\right)^{1/3}\right)^{5}\sqrt{8+6\left(-2\right)^{1/3}3^{2/3}}}+\frac{\left(2\,2^{2/3}+3\,3^{2/3}\right)\operatorname{arctanh}\left(\frac{2^{1/6}\left(3\,3^{1/3}+2^{1/3}x\right)}{\sqrt{-12+9\,2^{1/3}3^{2/3}}}\right)3^{5/6}}{78732\sqrt{-8+6\,2^{1/3}3^{2/3}}}$$

Result(type 7, 121 leaves):

$$\frac{\frac{73}{68364}x^{5}-\frac{1}{3798}x^{4}+\frac{227}{17091}x^{3}+\frac{4}{633}x^{2}-\frac{8}{5697}x+\frac{2}{211}}{x^{6}+18x^{4}+324x^{3}+108x^{2}+216}$$

$$+\frac{\left(\sum_{\_R=RootOf\left(\_Z^{6}+18\_Z^{4}+324\_Z^{3}+108\_Z^{2}+216\right)}\frac{\left(73\_R^{4}-36\_R^{3}+96\_R^{2}-216\_R+96\right)\ln\left(x-\_R\right)}{\_R^{5}+12\_R^{3}+162\_R^{2}+36\_R}\right)}{410184}$$

Problem 45: Result is not expressed in closed-form.

$$\int\frac{x^{4}}{\left(x^{6}+18x^{4}+324x^{3}+108x^{2}+216\right)^{2}}\,dx$$

Optimal(type 3, 588 leaves, 23 steps):

$$\frac{\left(-1\right)^{1/3}3^{2/3}\left(3\left(-3\right)^{1/3}2^{2/3}-2x\right)2^{1/3}}{34992\left(1+\left(-1\right)^{1/3}\right)^{4}\left(4-3\left(-3\right)^{2/3}2^{1/3}\right)\left(6-3\left(-3\right)^{1/3}2^{2/3}x+x^{2}\right)}-\frac{\left(-1\right)^{1/3}3^{2/3}\left(3\left(-2\right)^{2/3}3^{1/3}+2x\right)2^{1/3}}{157464\left(8+9\,I\,2^{1/3}3^{1/6}+3\,2^{1/3}3^{2/3}\right)\left(6+3\left(-2\right)^{2/3}3^{1/3}x+x^{2}\right)}$$

$$+\frac{-3\,3^{1/3}-2^{1/3}x}{52488\left(9\,2^{1/3}-4\,3^{1/3}\right)\left(6+3\,2^{2/3}3^{1/3}x+x^2\right)}+\frac{(-1)^{1/3}\arctan\left(\frac{3(-3)^{1/3}2^{2/3}-2x}{\sqrt{24-18(-3)^{2/3}2^{1/3}}}\right)2^{1/3}3^{1/6}}{4374\left(1+(-1)^{1/3}\right)^4\left(8-9\,I\,2^{1/3}3^{1/6}+3\,2^{1/3}3^{2/3}\right)^{3/2}}$$

$$-\frac{(-1)^{1/3}\arctan\left(\frac{3(-2)^{2/3}3^{1/3}+2x}{\sqrt{24+18(-2)^{1/3}3^{2/3}}}\right)2^{5/6}3^{1/6}}{17496\left(1-(-1)^{1/3}\right)^2\left(1+(-1)^{1/3}\right)^4\left(4+3(-2)^{1/3}3^{2/3}\right)^{3/2}}+\frac{\operatorname{arctanh}\left(\frac{2^{1/6}\left(3\,3^{1/3}+2^{1/3}x\right)}{\sqrt{-12+9\,2^{1/3}3^{2/3}}}\right)2^{5/6}3^{1/6}}{157464\left(-4+3\,2^{1/3}3^{2/3}\right)^{3/2}}$$

$$-\frac{\ln\left(6-3(-3)^{1/3}2^{2/3}x+x^2\right)2^{2/3}3^{1/3}}{209952\left(1+(-1)^{1/3}\right)^4}+\frac{I\ln\left(6+3(-2)^{2/3}3^{1/3}x+x^2\right)2^{2/3}3^{5/6}}{209952\left(1+(-1)^{1/3}\right)^5}-\frac{\ln\left(6+3\,2^{2/3}3^{1/3}x+x^2\right)2^{2/3}3^{1/3}}{1889568}$$

$$-\frac{I\arctan\left(\frac{2^{1/6}\left(3(-3)^{1/3}-2^{1/3}x\right)}{\sqrt{12-9(-3)^{2/3}2^{1/3}}}\right)2^{5/6}3^{2/3}}{34992\left(1+(-1)^{1/3}\right)^5\sqrt{4-3(-3)^{2/3}2^{1/3}}}-\frac{\arctan\left(\frac{3(-2)^{2/3}3^{1/3}+2x}{\sqrt{24+18(-2)^{1/3}3^{2/3}}}\right)\left(I+\sqrt{3}\right)2^{5/6}3^{2/3}}{69984\left(1+(-1)^{1/3}\right)^5\sqrt{4+3(-2)^{1/3}3^{2/3}}}$$

$$+\frac{\operatorname{arctanh}\left(\frac{2^{1/6}\left(3\,3^{1/3}+2^{1/3}x\right)}{\sqrt{-12+9\,2^{1/3}3^{2/3}}}\right)2^{5/6}3^{1/6}}{314928\sqrt{-4+3\,2^{1/3}3^{2/3}}}$$

Result(type 7, 121 leaves):

$$\frac{-\frac{1}{136728}x^5+\frac{1}{153819}x^4-\frac{1}{5697}x^3-\frac{1}{844}x^2+\frac{1}{3798}x-\frac{4}{17091}}{x^6+18x^4+324x^3+108x^2+216}$$

$$+\frac{\left(\sum_{\_R=RootOf(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216)}\frac{\left(-9\_R^4+16\_R^3-324\_R^2+2628\_R-324\right)\ln(x-\_R)}{\_R^5+12\_R^3+162\_R^2+36\_R}\right)}{7383312}$$

Problem 46: Result is not expressed in closed-form.

$$\int\frac{x^3}{\left(x^6+18x^4+324x^3+108x^2+216\right)^2}\,dx$$

Optimal(type 3, 601 leaves, 23 steps):

$$\frac{(-6)^{1/3}\left(2(-3)^{1/3}+9\,2^{1/3}\right)-3x}{157464\left(8-9\,I\,2^{1/3}3^{1/6}+3\,2^{1/3}3^{2/3}\right)\left(6-3(-3)^{1/3}2^{2/3}x+x^2\right)}+\frac{-(-6)^{1/3}\left(9(-2)^{1/3}+2\,3^{1/3}\right)-3x}{157464\left(8+9\,I\,2^{1/3}3^{1/6}+3\,2^{1/3}3^{2/3}\right)\left(6+3(-2)^{2/3}3^{1/3}x+x^2\right)}$$

$$+\frac{-2\,2^{1/3}+3\,6^{2/3}+3^{1/3}x}{104976\left(9\,2^{1/3}-4\,3^{1/3}\right)\left(6+3\,2^{2/3}3^{1/3}x+x^2\right)}-\frac{I\ln\left(6-3(-3)^{1/3}2^{2/3}x+x^2\right)2^{1/3}3^{1/6}}{139968\left(1+(-1)^{1/3}\right)^5}+\frac{\ln\left(6+3\,2^{2/3}3^{1/3}x+x^2\right)2^{1/3}3^{2/3}}{3779136}$$

$$+\frac{\arctan\left(\frac{3(-3)^{1/3}2^{2/3}-2x}{\sqrt{24-18(-3)^{2/3}2^{1/3}}}\right)\sqrt{3}}{78732\left(8-9\,I\,2^{1/3}3^{1/6}+3\,2^{1/3}3^{2/3}\right)^{3/2}}-\frac{\arctan\left(\frac{3(-2)^{2/3}3^{1/3}+2x}{\sqrt{24+18(-2)^{1/3}3^{2/3}}}\right)\sqrt{3}}{78732\left(8+9\,I\,2^{1/3}3^{1/6}+3\,2^{1/3}3^{2/3}\right)^{3/2}}$$

$$+\frac{\ln\left(6+3\,(-2)^{2/3}\,3^{1/3}\,x+x^2\right)\left(\mathrm{I}+\sqrt{3}\right)2^{1/3}\,3^{1/6}}{279936\,\left(1+(-1)^{1/3}\right)^5}-\frac{\operatorname{arctanh}\left(\frac{2^{1/6}\,(3\,3^{1/3}+2^{1/3}\,x)}{\sqrt{-12+9\,2^{1/3}\,3^{2/3}}}\right)\sqrt{6}}{314928\,\left(-4+3\,2^{1/3}\,3^{2/3}\right)^{3/2}}$$

$$-\frac{\arctan\left(\frac{3\,(-3)^{1/3}\,2^{2/3}-2\,x}{\sqrt{24-18\,(-3)^{2/3}\,2^{1/3}}}\right)\left(9\,\mathrm{I}-3^{1/3}\left(2\,\mathrm{I}\,2^{2/3}+9\,3^{1/6}+2\,2^{2/3}\sqrt{3}\right)\right)}{209952\,\left(1+(-1)^{1/3}\right)^5\sqrt{8-6\,(-3)^{2/3}\,2^{1/3}}}$$

$$+\frac{\left(9\,\mathrm{I}+3^{1/3}\left(4\,\mathrm{I}\,2^{2/3}-9\,3^{1/6}\right)\right)\arctan\left(\frac{3\,(-2)^{2/3}\,3^{1/3}+2\,x}{\sqrt{24+18\,(-2)^{1/3}\,3^{2/3}}}\right)}{209952\,\left(1+(-1)^{1/3}\right)^5\sqrt{8+6\,(-2)^{1/3}\,3^{2/3}}}+\frac{\left(2\,2^{2/3}-3\,3^{2/3}\right)\operatorname{arctanh}\left(\frac{2^{1/6}\,(3\,3^{1/3}+2^{1/3}\,x)}{\sqrt{-12+9\,2^{1/3}\,3^{2/3}}}\right)3^{5/6}}{2834352\,\sqrt{-8+6\,2^{1/3}\,3^{2/3}}}$$

Result(type 7, 121 leaves):

$$\frac{\frac{1}{922914}\,x^5-\frac{1}{136728}\,x^4+\frac{4}{153819}\,x^3+\frac{1}{5697}\,x^2-\frac{73}{68364}\,x+\frac{1}{3798}}{x^6+18\,x^4+324\,x^3+108\,x^2+216}$$

$$+\frac{\left(\sum_{\_R=RootOf(\_Z^6+18\,\_Z^4+324\,\_Z^3+108\,\_Z^2+216)}\frac{(2\,\_R^4-27\,\_R^3+72\,\_R^2-162\,\_R+1971)\ln(x-\_R)}{\_R^5+12\,\_R^3+162\,\_R^2+36\,\_R}\right)}{11074968}$$

Problem 51: Result more than twice size of optimal antiderivative.

$$\int x^7\left(d\,x^2+b\right)^7\left(3\,d\,x^2+b\right)\,\mathrm{d}x$$

Optimal(type 1, 14 leaves, 2 steps):

$$\frac{x^8\left(d\,x^2+b\right)^8}{8}$$

Result(type 1, 88 leaves):

$$\frac{1}{8}\,d^8\,x^{24}+b\,d^7\,x^{22}+\frac{7}{2}\,b^2\,d^6\,x^{20}+7\,b^3\,d^5\,x^{18}+\frac{35}{4}\,b^4\,d^4\,x^{16}+7\,b^5\,d^3\,x^{14}+\frac{7}{2}\,b^6\,d^2\,x^{12}+b^7\,d\,x^{10}+\frac{1}{8}\,b^8\,x^8$$

Problem 52: Result more than twice size of optimal antiderivative.

$$\int x^7\left(d\,x^2+c\,x\right)^7\left(3\,d\,x^2+2\,c\,x\right)\,\mathrm{d}x$$

Optimal(type 1, 12 leaves, 2 steps):

$$\frac{x^{16}\left(d\,x+c\right)^8}{8}$$

Result(type 1, 88 leaves):

$$\frac{1}{8}\,d^8\,x^{24}+c\,d^7\,x^{23}+\frac{7}{2}\,c^2\,d^6\,x^{22}+7\,c^3\,d^5\,x^{21}+\frac{35}{4}\,c^4\,d^4\,x^{20}+7\,c^5\,d^3\,x^{19}+\frac{7}{2}\,c^6\,d^2\,x^{18}+c^7\,d\,x^{17}+\frac{1}{8}\,c^8\,x^{16}$$

Problem 53: Result more than twice size of optimal antiderivative.

$$\int x^{15} (d x + c)^7 (3 d x + 2 c) \, dx$$

Optimal(type 1, 12 leaves, 1 step):

$$\frac{x^{16} (d x + c)^8}{8}$$

Result(type 1, 88 leaves):

$$\frac{1}{8} d^8 x^{24} + c d^7 x^{23} + \frac{7}{2} c^2 d^6 x^{22} + 7 c^3 d^5 x^{21} + \frac{35}{4} c^4 d^4 x^{20} + 7 c^5 d^3 x^{19} + \frac{7}{2} c^6 d^2 x^{18} + c^7 d x^{17} + \frac{1}{8} c^8 x^{16}$$

Problem 54: Result more than twice size of optimal antiderivative.

$$\int (b x + a) \left(1 + \left(c + a x + \frac{1}{2} b x^2\right)^4\right) dx$$

Optimal(type 1, 25 leaves, 2 steps):

$$a x + \frac{b x^2}{2} + \frac{\left(c + a x + \frac{1}{2} b x^2\right)^5}{5}$$

Result(type 1, 324 leaves):

$$\frac{b^5 x^{10}}{160} + \frac{a b^4 x^9}{16} + \frac{\left(\frac{a^2 b^3}{2} + b \left(\frac{(a^2 + b c) b^2}{2} + a^2 b^2\right)\right) x^8}{8} + \frac{\left(a \left(\frac{(a^2 + b c) b^2}{2} + a^2 b^2\right) + b \left(a c b^2 + 2 (a^2 + b c) a b\right)\right) x^7}{7}$$
$$+ \frac{\left(a \left(a c b^2 + 2 (a^2 + b c) a b\right) + b \left(\frac{c^2 b^2}{2} + 4 a^2 c b + (a^2 + b c)^2\right)\right) x^6}{6} + \frac{\left(a \left(\frac{c^2 b^2}{2} + 4 a^2 c b + (a^2 + b c)^2\right) + b \left(2 c^2 a b + 4 a c (a^2 + b c)\right)\right) x^5}{5}$$
$$+ \frac{\left(a \left(2 c^2 a b + 4 a c (a^2 + b c)\right) + b \left(2 c^2 (a^2 + b c) + 4 a^2 c^2\right)\right) x^4}{4} + \frac{\left(a \left(2 c^2 (a^2 + b c) + 4 a^2 c^2\right) + 4 b c^3 a\right) x^3}{3} + \frac{\left(4 a^2 c^3 + b (c^4 + 1)\right) x^2}{2} + a (c^4$$
$$+ 1) x$$

Problem 58: Result more than twice size of optimal antiderivative.

$$\int (1 + 2 x) (x^2 + x)^3 \left(-18 + 7 (x^2 + x)^3\right)^2 dx$$

Optimal(type 1, 31 leaves, ? steps):

$$81 x^4 (1 + x)^4 - 36 x^7 (1 + x)^7 + \frac{49 x^{10} (1 + x)^{10}}{10}$$

Result(type 1, 86 leaves):

$$\frac{49}{10} x^{20} + 49 x^{19} + \frac{441}{2} x^{18} + 588 x^{17} + 1029 x^{16} + \frac{6174}{5} x^{15} + 993 x^{14} + 336 x^{13} - \frac{1071}{2} x^{12} - 1211 x^{11} - \frac{12551}{10} x^{10} - 756 x^9 - 171 x^8 + 288 x^7 + 486 x^6$$
$$+ 324 x^5 + 81 x^4$$

Problem 69: Result is not expressed in closed-form.

$$\int \frac{x^3 (2x^3+3x^2+x+5)}{2x^4+x^3+5x^2+x+2}\,dx$$

Optimal(type 3, 217 leaves, 13 steps):

$$\frac{x^2(7-5I\sqrt{7})}{28}+\frac{x^3(7-5I\sqrt{7})}{42}+\frac{x^2(7+5I\sqrt{7})}{28}+\frac{x^3(7+5I\sqrt{7})}{42}-\frac{x(35-9I\sqrt{7})}{28}-\frac{x(35+9I\sqrt{7})}{28}$$
$$+\frac{3\ln\left(4+4x^2+x\left(1-I\sqrt{7}\right)\right)\left(7-11I\sqrt{7}\right)}{112}+\frac{3\ln\left(4+4x^2+x\left(1+I\sqrt{7}\right)\right)\left(7+11I\sqrt{7}\right)}{112}-\frac{11\arctan\left(\frac{1+8x+I\sqrt{7}}{\sqrt{70-2I\sqrt{7}}}\right)\left(9I-5\sqrt{7}\right)}{4\sqrt{490-14I\sqrt{7}}}$$
$$+\frac{11\arctan\left(\frac{1+8x-I\sqrt{7}}{\sqrt{70+2I\sqrt{7}}}\right)\left(9I+5\sqrt{7}\right)}{4\sqrt{490+14I\sqrt{7}}}$$

Result(type 7, 73 leaves):

$$\frac{x^3}{3}+\frac{x^2}{2}-\frac{5x}{2}+\frac{\left(\sum_{\_R=RootOf(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2)}\frac{(3\_R^3+19\_R^2+\_R+10)\ln(x-\_R)}{8\_R^3+3\_R^2+10\_R+1}\right)}{2}$$

Problem 90: Result more than twice size of optimal antiderivative.

$$\int \frac{cx^2+bx+a}{fx^4+ex^2+d}\,dx$$

Optimal(type 3, 171 leaves, 8 steps):

$$-\frac{b\,\mathrm{arctanh}\left(\frac{2fx^2+e}{\sqrt{-4df+e^2}}\right)}{\sqrt{-4df+e^2}}+\frac{\arctan\left(\frac{x\sqrt{2}\sqrt{f}}{\sqrt{e-\sqrt{-4df+e^2}}}\right)\left(c+\frac{2af-ec}{\sqrt{-4df+e^2}}\right)\sqrt{2}}{2\sqrt{f}\sqrt{e-\sqrt{-4df+e^2}}}+\frac{\arctan\left(\frac{x\sqrt{2}\sqrt{f}}{\sqrt{e+\sqrt{-4df+e^2}}}\right)\left(c+\frac{-2af+ec}{\sqrt{-4df+e^2}}\right)\sqrt{2}}{2\sqrt{f}\sqrt{e+\sqrt{-4df+e^2}}}$$

Result(type 3, 615 leaves):

$$\frac{\sqrt{-4df+e^2}\,b\ln\left(2fx^2+\sqrt{-4df+e^2}+e\right)}{2(4df-e^2)}+\frac{2f\sqrt{2}\arctan\left(\frac{fx\sqrt{2}}{\sqrt{\left(e+\sqrt{-4df+e^2}\right)f}}\right)cd}{(4df-e^2)\sqrt{\left(e+\sqrt{-4df+e^2}\right)f}}-\frac{\sqrt{2}\arctan\left(\frac{fx\sqrt{2}}{\sqrt{\left(e+\sqrt{-4df+e^2}\right)f}}\right)ce^2}{2(4df-e^2)\sqrt{\left(e+\sqrt{-4df+e^2}\right)f}}$$

$$+\frac{f\sqrt{-4df+e^2}\,\sqrt{2}\,\arctan\left(\frac{fx\sqrt{2}}{\sqrt{\left(e+\sqrt{-4df+e^2}\right)f}}\right)a}{(4df-e^2)\sqrt{\left(e+\sqrt{-4df+e^2}\right)f}}-\frac{\sqrt{-4df+e^2}\,\sqrt{2}\,\arctan\left(\frac{fx\sqrt{2}}{\sqrt{\left(e+\sqrt{-4df+e^2}\right)f}}\right)ec}{2\,(4df-e^2)\sqrt{\left(e+\sqrt{-4df+e^2}\right)f}}$$

$$-\frac{\sqrt{-4df+e^2}\,b\ln\left(-2fx^2+\sqrt{-4df+e^2}-e\right)}{2\,(4df-e^2)}-\frac{2f\sqrt{2}\,\operatorname{arctanh}\left(\frac{fx\sqrt{2}}{\sqrt{\left(\sqrt{-4df+e^2}-e\right)f}}\right)cd}{(4df-e^2)\sqrt{\left(\sqrt{-4df+e^2}-e\right)f}}+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{fx\sqrt{2}}{\sqrt{\left(\sqrt{-4df+e^2}-e\right)f}}\right)ce^2}{2\,(4df-e^2)\sqrt{\left(\sqrt{-4df+e^2}-e\right)f}}$$

$$+\frac{f\sqrt{-4df+e^2}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{fx\sqrt{2}}{\sqrt{\left(\sqrt{-4df+e^2}-e\right)f}}\right)a}{(4df-e^2)\sqrt{\left(\sqrt{-4df+e^2}-e\right)f}}-\frac{\sqrt{-4df+e^2}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{fx\sqrt{2}}{\sqrt{\left(\sqrt{-4df+e^2}-e\right)f}}\right)ec}{2\,(4df-e^2)\sqrt{\left(\sqrt{-4df+e^2}-e\right)f}}$$

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Problem 105: Result is not expressed in closed-form.
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$$\int \frac{x^2}{2-(-x^2+1)^4}\,dx$$

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Optimal(type 3, 103 leaves, 8 steps):
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$$-\frac{\mathrm{I}\operatorname{arctanh}\left(\frac{x}{\sqrt{1-\mathrm{I}2^{1/4}}}\right)\sqrt{1-\mathrm{I}2^{1/4}}\,2^{1/4}}{8}+\frac{\mathrm{I}\operatorname{arctanh}\left(\frac{x}{\sqrt{1+\mathrm{I}2^{1/4}}}\right)\sqrt{1+\mathrm{I}2^{1/4}}\,2^{1/4}}{8}-\frac{\arctan\left(\frac{x}{\sqrt{-1+2^{1/4}}}\right)\sqrt{-1+2^{1/4}}\,2^{1/4}}{8}$$

$$+\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{1+2^{1/4}}}\right)\sqrt{1+2^{1/4}}\,2^{1/4}}{8}$$

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Result(type 7, 55 leaves):
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$$-\frac{\left(\sum_{\_R=RootOf(\_Z^8-4\_Z^6+6\_Z^4-4\_Z^2-1)}\frac{\_R^2\ln(x-\_R)}{\_R^7-3\_R^5+3\_R^3-\_R}\right)}{8}$$

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Problem 106: Result is not expressed in closed-form.
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$$\int \frac{x^2}{2+(-x^2+1)^4}\,dx$$

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Optimal(type 3, 129 leaves, 8 steps):
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$$-\frac{(-1)^{1/4}\operatorname{arctanh}\left(\frac{x}{\sqrt{1-(-2)^{1/4}}}\right)\sqrt{1-(-2)^{1/4}}\,2^{1/4}}{8}+\frac{(-1)^{3/4}2^{1/4}\operatorname{arctanh}\left(\frac{x}{\sqrt{1+I(-2)^{1/4}}}\right)\sqrt{1+I(-2)^{1/4}}}{8}$$

$$+\frac{(-1)^{1/4}\operatorname{arctanh}\left(\frac{x}{\sqrt{1+(-2)^{1/4}}}\right)\sqrt{1+(-2)^{1/4}}\,2^{1/4}}{8}-\frac{I\operatorname{arctanh}\left(x\sqrt{\frac{1+I}{1+I+2^{3/4}}}\right)\left((-2)^{1/4}+\sqrt{2}\right)\sqrt{\frac{1+I}{1+I+2^{3/4}}}}{8}$$

Result(type 7, 55 leaves):

$$\frac{\left(\sum_{\_R=RootOf(\_Z^8-4\_Z^6+6\_Z^4-4\_Z^2+3)}\frac{\_R^2\ln(x-\_R)}{\_R^7-3\_R^5+3\_R^3-\_R}\right)}{8}$$

Problem 107: Result is not expressed in closed-form.

$$\int\frac{-x^2+1}{a+b\left(x^2-1\right)^4}\,dx$$

Optimal(type 3, 435 leaves, 17 steps):

$$-\frac{\arctan\left(\frac{b^{1/8}x}{\sqrt{(-a)^{1/4}-b^{1/4}}}\right)}{4b^{3/8}\sqrt{-a}\sqrt{(-a)^{1/4}-b^{1/4}}}+\frac{\operatorname{arctanh}\left(\frac{b^{1/8}x}{\sqrt{(-a)^{1/4}+b^{1/4}}}\right)}{4b^{3/8}\sqrt{-a}\sqrt{(-a)^{1/4}+b^{1/4}}}-\frac{\arctan\left(\frac{-b^{1/8}x\sqrt{2}+\sqrt{b^{1/4}+\sqrt{\sqrt{-a}+\sqrt{b}}}}{\sqrt{-b^{1/4}+\sqrt{\sqrt{-a}+\sqrt{b}}}}\right)\sqrt{-b^{1/4}+\sqrt{\sqrt{-a}+\sqrt{b}}}\,\sqrt{2}}{8b^{3/8}\sqrt{-a}\sqrt{\sqrt{-a}+\sqrt{b}}}$$

$$+\frac{\arctan\left(\frac{b^{1/8}x\sqrt{2}+\sqrt{b^{1/4}+\sqrt{\sqrt{-a}+\sqrt{b}}}}{\sqrt{-b^{1/4}+\sqrt{\sqrt{-a}+\sqrt{b}}}}\right)\sqrt{-b^{1/4}+\sqrt{\sqrt{-a}+\sqrt{b}}}\,\sqrt{2}}{8b^{3/8}\sqrt{-a}\sqrt{\sqrt{-a}+\sqrt{b}}}$$

$$+\frac{\ln\left(b^{1/4}x^2+\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt{2}\sqrt{b^{1/4}+\sqrt{\sqrt{-a}+\sqrt{b}}}\,b^{1/8}x\right)\sqrt{b^{1/4}+\sqrt{\sqrt{-a}+\sqrt{b}}}\,\sqrt{2}}{16b^{3/8}\sqrt{-a}\sqrt{\sqrt{-a}+\sqrt{b}}}$$

$$-\frac{\ln\left(b^{1/4}x^2+\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt{2}\sqrt{b^{1/4}+\sqrt{\sqrt{-a}+\sqrt{b}}}\,b^{1/8}x\right)\sqrt{b^{1/4}+\sqrt{\sqrt{-a}+\sqrt{b}}}\,\sqrt{2}}{16b^{3/8}\sqrt{-a}\sqrt{\sqrt{-a}+\sqrt{b}}}$$

Result(type 7, 68 leaves):

$$\frac{\sum_{\_R=RootOf(b\_Z^8-4b\_Z^6+6b\_Z^4-4b\_Z^2+a+b)}\frac{\left(-\_R^2+1\right)\ln(x-\_R)}{\_R^7-3\_R^5+3\_R^3-\_R}}{8b}$$

Problem 134: Result more than twice size of optimal antiderivative.

$$\int \frac{x^2}{1+(x^2-1)^2}\,dx$$

Optimal(type 3, 132 leaves, 10 steps):

$$-\frac{\arctan\left(\frac{-2x+\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)\sqrt{2+2\sqrt{2}}}{4}+\frac{\arctan\left(\frac{2x+\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)\sqrt{2+2\sqrt{2}}}{4}+\frac{\ln\left(x^2+\sqrt{2}-x\sqrt{2+2\sqrt{2}}\right)}{4\sqrt{2+2\sqrt{2}}}$$
$$-\frac{\ln\left(x^2+\sqrt{2}+x\sqrt{2+2\sqrt{2}}\right)}{4\sqrt{2+2\sqrt{2}}}$$

Result(type 3, 307 leaves):

$$\frac{\sqrt{2+2\sqrt{2}}\,\sqrt{2}\,\ln\left(x^2+\sqrt{2}-x\sqrt{2+2\sqrt{2}}\right)}{8}+\frac{\sqrt{2}\left(2+2\sqrt{2}\right)\arctan\left(\frac{2x-\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)}{4\sqrt{-2+2\sqrt{2}}}-\frac{\sqrt{2+2\sqrt{2}}\,\ln\left(x^2+\sqrt{2}-x\sqrt{2+2\sqrt{2}}\right)}{8}$$
$$-\frac{\left(2+2\sqrt{2}\right)\arctan\left(\frac{2x-\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)}{4\sqrt{-2+2\sqrt{2}}}-\frac{\sqrt{2+2\sqrt{2}}\,\sqrt{2}\,\ln\left(x^2+\sqrt{2}+x\sqrt{2+2\sqrt{2}}\right)}{8}+\frac{\sqrt{2}\left(2+2\sqrt{2}\right)\arctan\left(\frac{2x+\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)}{4\sqrt{-2+2\sqrt{2}}}$$
$$+\frac{\sqrt{2+2\sqrt{2}}\,\ln\left(x^2+\sqrt{2}+x\sqrt{2+2\sqrt{2}}\right)}{8}-\frac{\left(2+2\sqrt{2}\right)\arctan\left(\frac{2x+\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)}{4\sqrt{-2+2\sqrt{2}}}$$

Problem 135: Result is not expressed in closed-form.

$$\int\left(\frac{3\left(19x^3+120x^2+228x-47\right)}{\left(x^4+x+3\right)^4}+\frac{-8x^3-75x^2-320x+42}{\left(x^4+x+3\right)^3}+\frac{30x}{\left(x^4+x+3\right)^2}\right)dx$$

Optimal(type 1, 27 leaves, ? steps):

$$\frac{-5x^6+x^4+5x^2-3x+2}{\left(x^4+x+3\right)^3}$$

Result(type 7, 249 leaves):

$$\frac{\frac{377432}{195075}x^7-\frac{1404328}{195075}x^6+\frac{234517}{195075}x^5+\frac{660506}{195075}x^4-\frac{208792}{195075}x^3-\frac{13339729}{390150}x^2+\frac{89881}{13005}x+\frac{121303}{21675}}{\left(x^4+x+3\right)^2}$$

$$+\frac{\left(\sum_{\_R=RootOf(\_Z^4+\_Z+3)}\frac{(377432\_R^2-2808656\_R+703551)\ln(x-\_R)}{4\_R^3+1}\right)}{195075}+\frac{30\left(-\frac{16}{765}x^3+\frac{64}{765}x^2-\frac{1}{765}x-\frac{4}{255}\right)}{x^4+x+3}$$

$$+\frac{2\left(\sum_{\_R=RootOf(\_Z^4+\_Z+3)}\frac{(-16\_R^2+128\_R-3)\ln(x-\_R)}{4\_R^3+1}\right)}{51}+\frac{1}{(x^4+x+3)^3}\left(3\left(-\frac{255032}{585225}x^{11}+\frac{914728}{585225}x^{10}-\frac{226867}{585225}x^9-\frac{701338}{585225}x^8\right.\right.$$

$$\left.\left.+\frac{236024}{585225}x^7+\frac{13501313}{1170450}x^6-\frac{2360372}{585225}x^5-\frac{1873778}{585225}x^4+\frac{10935781}{1170450}x^3+\frac{3415123}{130050}x^2-\frac{62987}{7225}x-\frac{76253}{21675}\right)\right)$$

$$+\frac{\left(\sum_{\_R=RootOf(\_Z^4+\_Z+3)}\frac{(-255032\_R^2+1829456\_R-680601)\ln(x-\_R)}{4\_R^3+1}\right)}{195075}$$

Problem 136: Result more than twice size of optimal antiderivative.

$$\int\left(\frac{-30x^5+4x^3+10x-3}{(x^4+x+3)^3}-\frac{3(4x^3+1)(-5x^6+x^4+5x^2-3x+2)}{(x^4+x+3)^4}\right)dx$$

Optimal(type 1, 27 leaves, ? steps):

$$\frac{-5x^6+x^4+5x^2-3x+2}{(x^4+x+3)^3}$$

Result(type 1, 111 leaves):

$$-\frac{-\frac{34568}{195075}x^7+\frac{73672}{195075}x^6+\frac{15392}{195075}x^5-\frac{60494}{195075}x^4-\frac{68792}{195075}x^3-\frac{583927}{195075}x^2+\frac{3356}{13005}x-\frac{2069}{43350}}{(x^4+x+3)^2}+\frac{1}{(x^4+x+3)^3}\left(3\left(-\frac{34568}{585225}x^{11}+\frac{73672}{585225}x^{10}\right.\right.$$

$$\left.\left.+\frac{15392}{585225}x^9-\frac{95062}{585225}x^8-\frac{98824}{585225}x^7-\frac{1322894}{585225}x^6+\frac{36022}{585225}x^5-\frac{129019}{1170450}x^4-\frac{790303}{585225}x^3-\frac{80674}{65025}x^2-\frac{10951}{14450}x+\frac{26831}{43350}\right)\right)$$

Test results for the 266 problems in "1.3.2 Algebraic functions.txt"

Problem 3: Unable to integrate problem.

$$\int\frac{1}{(2^{2/3}a^{1/3}-b^{1/3}x)\sqrt{bx^3-a}}\,dx$$

Optimal(type 4, 216 leaves, 4 steps):

$$-\frac{2\,\mathrm{arctanh}\left(\frac{a^{1/6}(a^{1/3}-2^{1/3}b^{1/3}x)\sqrt{3}}{\sqrt{bx^3-a}}\right)\sqrt{3}}{9b^{1/3}\sqrt{a}}$$

$$-\frac{2\,2^{1/3}\left(a^{1/3}-b^{1/3}x\right)\mathrm{EllipticF}\left(\frac{-b^{1/3}x+a^{1/3}\left(1+\sqrt{3}\right)}{-b^{1/3}x+a^{1/3}\left(1-\sqrt{3}\right)},2\,\mathrm{I}-\mathrm{I}\sqrt{3}\right)\sqrt{\frac{a^{2/3}+a^{1/3}b^{1/3}x+b^{2/3}x^{2}}{\left(-b^{1/3}x+a^{1/3}\left(1-\sqrt{3}\right)\right)^{2}}}\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)3^{3/4}}{9\,a^{1/3}b^{1/3}\sqrt{b\,x^{3}-a}\sqrt{-\frac{a^{1/3}\left(a^{1/3}-b^{1/3}x\right)}{\left(-b^{1/3}x+a^{1/3}\left(1-\sqrt{3}\right)\right)^{2}}}}$$

Result(type 8, 30 leaves):

$$\int\frac{1}{\left(2^{2/3}a^{1/3}-b^{1/3}x\right)\sqrt{b\,x^{3}-a}}\,dx$$

Problem 8: Unable to integrate problem.

$$\int\frac{1}{(d\,x+c)\,\left(d^{3}x^{3}-c^{3}\right)^{1/3}}\,dx$$

Optimal(type 3, 116 leaves, 1 step):

$$\frac{\ln\left((-d\,x+c)\,(d\,x+c)^{2}\right)2^{2/3}}{8\,c\,d}-\frac{3\ln\left(d\,(-d\,x+c)+2^{2/3}d\left(d^{3}x^{3}-c^{3}\right)^{1/3}\right)2^{2/3}}{8\,c\,d}+\frac{\arctan\left(\frac{\left(1-\frac{2^{1/3}(-d\,x+c)}{\left(d^{3}x^{3}-c^{3}\right)^{1/3}}\right)\sqrt{3}}{3}\right)\sqrt{3}\,2^{2/3}}{4\,c\,d}$$

Result(type 8, 25 leaves):

$$\int\frac{1}{(d\,x+c)\,\left(d^{3}x^{3}-c^{3}\right)^{1/3}}\,dx$$

Problem 9: Unable to integrate problem.

$$\int\frac{1}{(d\,x+c)\,\left(d^{3}x^{3}+2\,c^{3}\right)^{1/3}}\,dx$$

Optimal(type 3, 162 leaves, 3 steps):

$$-\frac{\ln(d\,x+c)}{2\,c\,d}-\frac{\ln\left(-d\,x+\left(d^{3}x^{3}+2\,c^{3}\right)^{1/3}\right)}{4\,c\,d}+\frac{3\ln\left(d\,(d\,x+2\,c)-d\left(d^{3}x^{3}+2\,c^{3}\right)^{1/3}\right)}{4\,c\,d}+\frac{\arctan\left(\frac{\left(1+\frac{2\,d\,x}{\left(d^{3}x^{3}+2\,c^{3}\right)^{1/3}}\right)\sqrt{3}}{3}\right)\sqrt{3}}{6\,c\,d}$$
$$-\frac{\arctan\left(\frac{\left(1+\frac{2\,(d\,x+2\,c)}{\left(d^{3}x^{3}+2\,c^{3}\right)^{1/3}}\right)\sqrt{3}}{3}\right)\sqrt{3}}{2\,c\,d}$$

Result(type 8, 25 leaves):

$$\int \frac{1}{(d x+c)\,(d^3 x^3+2 c^3)^{1/3}}\,dx$$

Problem 10: Unable to integrate problem.

$$\int (d x+c)^4\,(b x^3+a)^{1/3}\,dx$$

Optimal(type 5, 317 leaves, 11 steps):

$$\frac{3 a c^2 d^2 (b x^3+a)^{1/3}}{2 b}+\frac{a d^4 x^2 (b x^3+a)^{1/3}}{18 b}+\frac{(b x^3+a)^{1/3} (5 d^4 x^5+24 c d^3 x^4+45 c^2 d^2 x^3+40 c^3 d x^2+15 c^4 x)}{30}$$
$$+\frac{a c^4 x \left(1+\frac{b x^3}{a}\right)^{2/3} \mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{2}{3}\right],\left[\frac{4}{3}\right],-\frac{b x^3}{a}\right)}{2 (b x^3+a)^{2/3}}+\frac{a c d^3 x^4 \left(1+\frac{b x^3}{a}\right)^{2/3} \mathrm{hypergeom}\left(\left[\frac{2}{3},\frac{4}{3}\right],\left[\frac{7}{3}\right],-\frac{b x^3}{a}\right)}{5 (b x^3+a)^{2/3}}$$
$$-\frac{2 a c^3 d \ln\left(b^{1/3} x-(b x^3+a)^{1/3}\right)}{3 b^{2/3}}+\frac{a^2 d^4 \ln\left(b^{1/3} x-(b x^3+a)^{1/3}\right)}{18 b^{5/3}}-\frac{4 a c^3 d \arctan\left(\frac{\left(1+\frac{2 b^{1/3} x}{(b x^3+a)^{1/3}}\right)\sqrt{3}}{3}\right)\sqrt{3}}{9 b^{2/3}}$$
$$+\frac{a^2 d^4 \arctan\left(\frac{\left(1+\frac{2 b^{1/3} x}{(b x^3+a)^{1/3}}\right)\sqrt{3}}{3}\right)\sqrt{3}}{27 b^{5/3}}$$

Result(type 8, 159 leaves):

$$\frac{(15 d^4 x^5 b+72 c d^3 x^4 b+135 c^2 d^2 x^3 b+5 a d^4 x^2+120 b c^3 d x^2+36 a c d^3 x+45 b c^4 x+135 c^2 d^2 a)\,(b x^3+a)^{1/3}}{90 b}$$
$$+\frac{\left(\int -\frac{a (10 a d^4 x-120 b c^3 d x+36 a c d^3-45 b c^4)}{90 b \left((b x^3+a)^2\right)^{1/3}}\,dx\right)\left((b x^3+a)^2\right)^{1/3}}{(b x^3+a)^{2/3}}$$

Problem 11: Unable to integrate problem.

$$\int \frac{(d x+c)^4}{(b x^3+a)^{1/3}}\,dx$$

Optimal(type 5, 251 leaves, 10 steps):

$$\frac{3 c^2 d^2 (b x^3+a)^{2/3}}{b}+\frac{4 c d^3 x (b x^3+a)^{2/3}}{3 b}+\frac{2 c^3 d x^2 \left(1+\frac{b x^3}{a}\right)^{1/3} \mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{2}{3}\right],\left[\frac{5}{3}\right],-\frac{b x^3}{a}\right)}{(b x^3+a)^{1/3}}$$

$$+\frac{d^4 x^5 \left(1+\frac{b x^3}{a}\right)^{1/3} \operatorname{hypergeom}\left(\left[\frac{1}{3}, \frac{5}{3}\right], \left[\frac{8}{3}\right], -\frac{b x^3}{a}\right)}{5 \left(b x^3+a\right)^{1/3}} - \frac{c^4 \ln\left(-b^{1/3} x+\left(b x^3+a\right)^{1/3}\right)}{2 b^{1/3}} + \frac{2 a c d^3 \ln\left(-b^{1/3} x+\left(b x^3+a\right)^{1/3}\right)}{3 b^{4/3}}$$

$$+\frac{c^4 \arctan\left(\frac{\left(1+\frac{2 b^{1/3} x}{\left(b x^3+a\right)^{1/3}}\right)\sqrt{3}}{3}\right)\sqrt{3}}{3 b^{1/3}} - \frac{4 a c d^3 \arctan\left(\frac{\left(1+\frac{2 b^{1/3} x}{\left(b x^3+a\right)^{1/3}}\right)\sqrt{3}}{3}\right)\sqrt{3}}{9 b^{4/3}}$$

Result(type 8, 82 leaves):

$$\frac{d^2 \left(3 d^2 x^2+16 c d x+36 c^2\right) \left(b x^3+a\right)^{2/3}}{12 b} + \int -\frac{3 a d^4 x-24 b c^3 d x+8 a c d^3-6 b c^4}{6 b \left(b x^3+a\right)^{1/3}}\, dx$$

Problem 12: Unable to integrate problem.

$$\int \frac{1}{\left(d x+c\right)^3 \left(b x^3+a\right)^{1/3}}\, dx$$

Optimal(type 6, 1346 leaves, 32 steps):

$$\frac{3 c^4 d^2 \left(b x^3+a\right)^{2/3}}{2 \left(-a d^3+b c^3\right) \left(d^3 x^3+c^3\right)^2} - \frac{3 c^3 d^3 x \left(b x^3+a\right)^{2/3}}{2 \left(-a d^3+b c^3\right) \left(d^3 x^3+c^3\right)^2} + \frac{4 b c^4 d^2 \left(b x^3+a\right)^{2/3}}{3 \left(-a d^3+b c^3\right)^2 \left(d^3 x^3+c^3\right)} - \frac{c d^2 \left(-3 a d^3+b c^3\right) \left(b x^3+a\right)^{2/3}}{3 \left(-a d^3+b c^3\right)^2 \left(d^3 x^3+c^3\right)}$$

$$+\frac{d^3 \left(-7 a d^3+3 b c^3\right) x \left(b x^3+a\right)^{2/3}}{18 \left(-a d^3+b c^3\right)^2 \left(d^3 x^3+c^3\right)} - \frac{d^3 \left(-5 a d^3+9 b c^3\right) x \left(b x^3+a\right)^{2/3}}{18 \left(-a d^3+b c^3\right)^2 \left(d^3 x^3+c^3\right)} - \frac{7 d^3 \left(a d^3+3 b c^3\right) x \left(b x^3+a\right)^{2/3}}{18 \left(-a d^3+b c^3\right)^2 \left(d^3 x^3+c^3\right)}$$

$$-\frac{3 d x^2 \left(1+\frac{b x^3}{a}\right)^{1/3} \mathit{AppellF1}\left(\frac{2}{3}, \frac{1}{3}, 3, \frac{5}{3}, -\frac{b x^3}{a}, -\frac{d^3 x^3}{c^3}\right)}{2 c^4 \left(b x^3+a\right)^{1/3}} + \frac{6 d^4 x^5 \left(1+\frac{b x^3}{a}\right)^{1/3} \mathit{AppellF1}\left(\frac{5}{3}, \frac{1}{3}, 3, \frac{8}{3}, -\frac{b x^3}{a}, -\frac{d^3 x^3}{c^3}\right)}{5 c^7 \left(b x^3+a\right)^{1/3}}$$

$$+\frac{2 b^2 c^4 \ln\left(d^3 x^3+c^3\right)}{9 \left(-a d^3+b c^3\right)^{7/3}} + \frac{a^2 d^6 \ln\left(d^3 x^3+c^3\right)}{27 c^2 \left(-a d^3+b c^3\right)^{7/3}} - \frac{b c \left(-3 a d^3+b c^3\right) \ln\left(d^3 x^3+c^3\right)}{18 \left(-a d^3+b c^3\right)^{7/3}} + \frac{7 a d^3 \left(-a d^3+3 b c^3\right) \ln\left(d^3 x^3+c^3\right)}{54 c^2 \left(-a d^3+b c^3\right)^{7/3}}$$

$$+\frac{\left(5 a^2 d^6-12 a b c^3 d^3+9 b^2 c^6\right) \ln\left(d^3 x^3+c^3\right)}{54 c^2 \left(-a d^3+b c^3\right)^{7/3}} - \frac{a^2 d^6 \ln\left(\frac{\left(-a d^3+b c^3\right)^{1/3} x}{c} - \left(b x^3+a\right)^{1/3}\right)}{9 c^2 \left(-a d^3+b c^3\right)^{7/3}}$$

$$-\frac{7 a d^3 \left(-a d^3+3 b c^3\right) \ln\left(\frac{\left(-a d^3+b c^3\right)^{1/3} x}{c} - \left(b x^3+a\right)^{1/3}\right)}{18 c^2 \left(-a d^3+b c^3\right)^{7/3}} - \frac{\left(5 a^2 d^6-12 a b c^3 d^3+9 b^2 c^6\right) \ln\left(\frac{\left(-a d^3+b c^3\right)^{1/3} x}{c} - \left(b x^3+a\right)^{1/3}\right)}{18 c^2 \left(-a d^3+b c^3\right)^{7/3}}$$

$$-\frac{2 b^2 c^4 \ln\left(\left(-a d^3+b c^3\right)^{1/3} + d \left(b x^3+a\right)^{1/3}\right)}{3 \left(-a d^3+b c^3\right)^{7/3}} + \frac{b c \left(-3 a d^3+b c^3\right) \ln\left(\left(-a d^3+b c^3\right)^{1/3} + d \left(b x^3+a\right)^{1/3}\right)}{6 \left(-a d^3+b c^3\right)^{7/3}}$$

$$+\frac{2 a^2 d^6 \arctan\left(\frac{\left(1+\frac{2 (-a d^3+b c^3)^{1/3} x}{c (b x^3+a)^{1/3}}\right)\sqrt{3}}{3}\right)\sqrt{3}}{27 c^2 (-a d^3+b c^3)^{7/3}}+\frac{7 a d^3 (-a d^3+3 b c^3) \arctan\left(\frac{\left(1+\frac{2 (-a d^3+b c^3)^{1/3} x}{c (b x^3+a)^{1/3}}\right)\sqrt{3}}{3}\right)\sqrt{3}}{27 c^2 (-a d^3+b c^3)^{7/3}}$$
$$+\frac{(5 a^2 d^6-12 a b c^3 d^3+9 b^2 c^6) \arctan\left(\frac{\left(1+\frac{2 (-a d^3+b c^3)^{1/3} x}{c (b x^3+a)^{1/3}}\right)\sqrt{3}}{3}\right)\sqrt{3}}{27 c^2 (-a d^3+b c^3)^{7/3}}-\frac{4 b^2 c^4 \arctan\left(\frac{\left(1-\frac{2 d (b x^3+a)^{1/3}}{(-a d^3+b c^3)^{1/3}}\right)\sqrt{3}}{3}\right)\sqrt{3}}{9 (-a d^3+b c^3)^{7/3}}$$
$$+\frac{b c (-3 a d^3+b c^3) \arctan\left(\frac{\left(1-\frac{2 d (b x^3+a)^{1/3}}{(-a d^3+b c^3)^{1/3}}\right)\sqrt{3}}{3}\right)\sqrt{3}}{9 (-a d^3+b c^3)^{7/3}}$$

Result(type 8, 19 leaves):

$$\int \frac{1}{(d x+c)^3 (b x^3+a)^{1/3}}\, dx$$

Problem 13: Unable to integrate problem.

$$\int \frac{d x+c}{(b x^3+a)^{2/3}}\, dx$$

Optimal(type 5, 96 leaves, 5 steps):

$$\frac{c x \left(1+\frac{b x^3}{a}\right)^{2/3} \mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{2}{3}\right],\left[\frac{4}{3}\right],-\frac{b x^3}{a}\right)}{(b x^3+a)^{2/3}}-\frac{d \ln\left(b^{1/3} x-(b x^3+a)^{1/3}\right)}{2 b^{2/3}}-\frac{d \arctan\left(\frac{\left(1+\frac{2 b^{1/3} x}{(b x^3+a)^{1/3}}\right)\sqrt{3}}{3}\right)\sqrt{3}}{3 b^{2/3}}$$

Result(type 8, 17 leaves):

$$\int \frac{d x+c}{(b x^3+a)^{2/3}}\, dx$$

Problem 14: Unable to integrate problem.

$$\int \frac{1}{(d x+c)^2 (b x^3+a)^{2/3}}\, dx$$

Optimal(type 6, 669 leaves, 18 steps):

$$\frac{c^2 d^2 (b x^3+a)^{1/3}}{(-a d^3+b c^3)(d^3 x^3+c^3)}+\frac{d^4 x^2 (b x^3+a)^{1/3}}{(-a d^3+b c^3)(d^3 x^3+c^3)}+\frac{x\left(1+\frac{b x^3}{a}\right)^{2/3} \mathit{AppellF1}\left(\frac{1}{3},\frac{2}{3},2,\frac{4}{3},-\frac{b x^3}{a},-\frac{d^3 x^3}{c^3}\right)}{c^2 (b x^3+a)^{2/3}}$$
$$-\frac{d^3 x^4\left(1+\frac{b x^3}{a}\right)^{2/3} \mathit{AppellF1}\left(\frac{4}{3},\frac{2}{3},2,\frac{7}{3},-\frac{b x^3}{a},-\frac{d^3 x^3}{c^3}\right)}{2 c^5 (b x^3+a)^{2/3}}-\frac{b c^2 d \ln(d^3 x^3+c^3)}{3(-a d^3+b c^3)^{5/3}}-\frac{a d^4 \ln(d^3 x^3+c^3)}{9 c(-a d^3+b c^3)^{5/3}}-\frac{d(-a d^3+3 b c^3)\ln(d^3 x^3+c^3)}{9 c(-a d^3+b c^3)^{5/3}}$$
$$+\frac{a d^4 \ln\left(\frac{(-a d^3+b c^3)^{1/3} x}{c}-(b x^3+a)^{1/3}\right)}{3 c(-a d^3+b c^3)^{5/3}}+\frac{d(-a d^3+3 b c^3)\ln\left(\frac{(-a d^3+b c^3)^{1/3} x}{c}-(b x^3+a)^{1/3}\right)}{3 c(-a d^3+b c^3)^{5/3}}$$
$$+\frac{b c^2 d \ln\left((-a d^3+b c^3)^{1/3}+d(b x^3+a)^{1/3}\right)}{(-a d^3+b c^3)^{5/3}}+\frac{2 a d^4 \arctan\left(\frac{\left(1+\frac{2(-a d^3+b c^3)^{1/3} x}{c(b x^3+a)^{1/3}}\right)\sqrt{3}}{3}\right)\sqrt{3}}{9 c(-a d^3+b c^3)^{5/3}}$$
$$+\frac{2 d(-a d^3+3 b c^3)\arctan\left(\frac{\left(1+\frac{2(-a d^3+b c^3)^{1/3} x}{c(b x^3+a)^{1/3}}\right)\sqrt{3}}{3}\right)\sqrt{3}}{9 c(-a d^3+b c^3)^{5/3}}-\frac{2 b c^2 d \arctan\left(\frac{\left(1-\frac{2 d(b x^3+a)^{1/3}}{(-a d^3+b c^3)^{1/3}}\right)\sqrt{3}}{3}\right)\sqrt{3}}{3(-a d^3+b c^3)^{5/3}}$$

Result(type 8, 19 leaves):

$$\int \frac{1}{(d x+c)^2 (b x^3+a)^{2/3}}\, dx$$

Problem 15: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{2^{2/3}+2x}{(2^{2/3}-x)\sqrt{x^3-1}}\, dx$$

Optimal(type 3, 28 leaves, 2 steps):

$$-\frac{2\, 2^{2/3} \operatorname{arctanh}\left(\frac{(1-2^{1/3} x)\sqrt{3}}{\sqrt{x^3-1}}\right)\sqrt{3}}{3}$$

Result(type 4, 261 leaves):

$$-\frac{4\left(-\frac{3}{2}-\frac{I\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{I\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{I\sqrt{3}}{2}}{\frac{3}{2}-\frac{I\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{I\sqrt{3}}{2}}{\frac{3}{2}+\frac{I\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{I\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{I\sqrt{3}}{2}}{\frac{3}{2}-\frac{I\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}$$

$$-\frac{1}{\sqrt{x^3-1}\,(-2^{2/3}+1)}\left(6\,2^{2/3}\left(-\frac{3}{2}-\frac{I\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{I\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{I\sqrt{3}}{2}}{\frac{3}{2}-\frac{I\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{I\sqrt{3}}{2}}{\frac{3}{2}+\frac{I\sqrt{3}}{2}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{I\sqrt{3}}{2}}},\frac{\frac{3}{2}+\frac{I\sqrt{3}}{2}}{-2^{2/3}+1},\sqrt{\frac{\frac{3}{2}+\frac{I\sqrt{3}}{2}}{\frac{3}{2}-\frac{I\sqrt{3}}{2}}}\right)\right)$$

Problem 16: Unable to integrate problem.

$$\int\frac{2^{2/3}a^{1/3}-2\,b^{1/3}x}{(2^{2/3}a^{1/3}+b^{1/3}x)\sqrt{b\,x^3+a}}\,dx$$

Optimal(type 3, 43 leaves, 2 steps):

$$\frac{2\,2^{2/3}\arctan\left(\frac{a^{1/6}(a^{1/3}+2^{1/3}b^{1/3}x)\sqrt{3}}{\sqrt{b\,x^3+a}}\right)\sqrt{3}}{3\,a^{1/6}b^{1/3}}$$

Result(type 8, 41 leaves):

$$\int\frac{2^{2/3}a^{1/3}-2\,b^{1/3}x}{(2^{2/3}a^{1/3}+b^{1/3}x)\sqrt{b\,x^3+a}}\,dx$$

Problem 17: Unable to integrate problem.

$$\int\frac{2^{2/3}a^{1/3}+2\,b^{1/3}x}{(2^{2/3}a^{1/3}-b^{1/3}x)\sqrt{b\,x^3-a}}\,dx$$

Optimal(type 3, 46 leaves, 2 steps):

$$-\frac{2\,2^{2/3}\operatorname{arctanh}\left(\frac{a^{1/6}(a^{1/3}-2^{1/3}b^{1/3}x)\sqrt{3}}{\sqrt{b\,x^3-a}}\right)\sqrt{3}}{3\,a^{1/6}b^{1/3}}$$

Result(type 8, 44 leaves):

$$\int \frac{2^{2/3} a^{1/3} + 2 b^{1/3} x}{\left(2^{2/3} a^{1/3} - b^{1/3} x\right) \sqrt{b x^3 - a}}\, dx$$

Problem 18: Result more than twice size of optimal antiderivative.

$$\int \frac{f x + e}{\left(2^{2/3} + x\right) \sqrt{x^3 + 1}}\, dx$$

Optimal(type 4, 126 leaves, 4 steps):

$$\frac{2 \left(e - 2^{2/3} f\right) \arctan\left(\frac{\left(1 + 2^{1/3} x\right) \sqrt{3}}{\sqrt{x^3 + 1}}\right) \sqrt{3}}{9} + \frac{2 \left(2^{1/3} e + f\right) (1 + x) \operatorname{EllipticF}\left(\frac{1 + x - \sqrt{3}}{1 + x + \sqrt{3}}, \mathrm{I}\sqrt{3} + 2\mathrm{I}\right) \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}\right) \sqrt{\frac{x^2 - x + 1}{\left(1 + x + \sqrt{3}\right)^2}}\, 3^{3/4}}{9 \sqrt{x^3 + 1} \sqrt{\frac{1 + x}{\left(1 + x + \sqrt{3}\right)^2}}}$$

Result(type 4, 263 leaves):

$$\frac{2 f \left(\frac{3}{2} - \frac{\mathrm{I}\sqrt{3}}{2}\right) \sqrt{\frac{1 + x}{\frac{3}{2} - \frac{\mathrm{I}\sqrt{3}}{2}}} \sqrt{\frac{x - \frac{1}{2} - \frac{\mathrm{I}\sqrt{3}}{2}}{-\frac{3}{2} - \frac{\mathrm{I}\sqrt{3}}{2}}} \sqrt{\frac{x - \frac{1}{2} + \frac{\mathrm{I}\sqrt{3}}{2}}{-\frac{3}{2} + \frac{\mathrm{I}\sqrt{3}}{2}}} \operatorname{EllipticF}\left(\sqrt{\frac{1 + x}{\frac{3}{2} - \frac{\mathrm{I}\sqrt{3}}{2}}}, \sqrt{\frac{-\frac{3}{2} + \frac{\mathrm{I}\sqrt{3}}{2}}{-\frac{3}{2} - \frac{\mathrm{I}\sqrt{3}}{2}}}\right)}{\sqrt{x^3 + 1}}$$

$$+ \frac{1}{\sqrt{x^3 + 1} \left(2^{2/3} - 1\right)} \left(2 \left(e - 2^{2/3} f\right) \left(\frac{3}{2} - \frac{\mathrm{I}\sqrt{3}}{2}\right) \sqrt{\frac{1 + x}{\frac{3}{2} - \frac{\mathrm{I}\sqrt{3}}{2}}} \sqrt{\frac{x - \frac{1}{2} - \frac{\mathrm{I}\sqrt{3}}{2}}{-\frac{3}{2} - \frac{\mathrm{I}\sqrt{3}}{2}}} \sqrt{\frac{x - \frac{1}{2} + \frac{\mathrm{I}\sqrt{3}}{2}}{-\frac{3}{2} + \frac{\mathrm{I}\sqrt{3}}{2}}} \operatorname{EllipticPi}\left(\sqrt{\frac{1 + x}{\frac{3}{2} - \frac{\mathrm{I}\sqrt{3}}{2}}},\right.\right.$$

$$\left.\left.\frac{-\frac{3}{2} + \frac{\mathrm{I}\sqrt{3}}{2}}{2^{2/3} - 1}, \sqrt{\frac{-\frac{3}{2} + \frac{\mathrm{I}\sqrt{3}}{2}}{-\frac{3}{2} - \frac{\mathrm{I}\sqrt{3}}{2}}}\right)\right)$$

Problem 20: Result more than twice size of optimal antiderivative.

$$\int \frac{f x + e}{(d x + c) \sqrt{4 d^3 x^3 + c^3}}\, dx$$

Optimal(type 4, 216 leaves, 4 steps):

$$\frac{2\left(-cf+de\right)\arctan\left(\frac{(2dx+c)\sqrt{3}\sqrt{c}}{\sqrt{4d^3x^3+c^3}}\right)\sqrt{3}}{9c^{3/2}d^2}$$

$$+\frac{2^{1/3}(cf+2de)\left(c+2^{2/3}dx\right)\mathrm{EllipticF}\left(\frac{2^{2/3}dx+c\left(1-\sqrt{3}\right)}{2^{2/3}dx+c\left(1+\sqrt{3}\right)},\mathrm{I}\sqrt{3}+2\mathrm{I}\right)\left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)\sqrt{\frac{c^2-2^{2/3}cdx+2\,2^{1/3}d^2x^2}{\left(2^{2/3}dx+c\left(1+\sqrt{3}\right)\right)^2}}\;3^{3/4}}{9cd^2\sqrt{4d^3x^3+c^3}\sqrt{\frac{c\left(c+2^{2/3}dx\right)}{\left(2^{2/3}dx+c\left(1+\sqrt{3}\right)\right)^2}}}$$

Result(type 4, 899 leaves):

$$\frac{1}{d\sqrt{4d^3x^3+c^3}}\left(2f\left(\frac{\left(\frac{2^{1/3}}{4}-\frac{\mathrm{I}\sqrt{3}\,2^{1/3}}{4}\right)c}{d}\right.\right.$$

$$\left.-\frac{\left(\frac{2^{1/3}}{4}+\frac{\mathrm{I}\sqrt{3}\,2^{1/3}}{4}\right)c}{d}\right)$$

$$\sqrt{\frac{x-\frac{\left(\frac{2^{1/3}}{4}+\frac{\mathrm{I}\sqrt{3}\,2^{1/3}}{4}\right)c}{d}}{\frac{\left(\frac{2^{1/3}}{4}-\frac{\mathrm{I}\sqrt{3}\,2^{1/3}}{4}\right)c}{d}-\frac{\left(\frac{2^{1/3}}{4}+\frac{\mathrm{I}\sqrt{3}\,2^{1/3}}{4}\right)c}{d}}}\sqrt{\frac{x+\frac{2^{1/3}c}{2d}}{\frac{\left(\frac{2^{1/3}}{4}+\frac{\mathrm{I}\sqrt{3}\,2^{1/3}}{4}\right)c}{d}+\frac{2^{1/3}c}{2d}}}$$

$$\sqrt{\frac{x-\frac{\left(\frac{2^{1/3}}{4}-\frac{\mathrm{I}\sqrt{3}\,2^{1/3}}{4}\right)c}{d}}{\frac{\left(\frac{2^{1/3}}{4}+\frac{\mathrm{I}\sqrt{3}\,2^{1/3}}{4}\right)c}{d}-\frac{\left(\frac{2^{1/3}}{4}-\frac{\mathrm{I}\sqrt{3}\,2^{1/3}}{4}\right)c}{d}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x-\frac{\left(\frac{2^{1/3}}{4}+\frac{\mathrm{I}\sqrt{3}\,2^{1/3}}{4}\right)c}{d}}{\frac{\left(\frac{2^{1/3}}{4}-\frac{\mathrm{I}\sqrt{3}\,2^{1/3}}{4}\right)c}{d}-\frac{\left(\frac{2^{1/3}}{4}+\frac{\mathrm{I}\sqrt{3}\,2^{1/3}}{4}\right)c}{d}}},\right.$$

$$\sqrt{\frac{\frac{\left(\frac{2^{1/3}}{4}+\frac{I\sqrt{3}\,2^{1/3}}{4}\right)c}{d}-\frac{\left(\frac{2^{1/3}}{4}-\frac{I\sqrt{3}\,2^{1/3}}{4}\right)c}{d}}{\frac{\left(\frac{2^{1/3}}{4}+\frac{I\sqrt{3}\,2^{1/3}}{4}\right)c}{d}+\frac{2^{1/3}c}{2d}}}\Bigg)\Bigg)+\frac{1}{d^2\sqrt{4d^3x^3+c^3}\left(\frac{\left(\frac{2^{1/3}}{4}+\frac{I\sqrt{3}\,2^{1/3}}{4}\right)c}{d}+\frac{c}{d}\right)}\Bigg(2\,(-cf$$

$$+d\,e)\left(\frac{\left(\frac{2^{1/3}}{4}-\frac{I\sqrt{3}\,2^{1/3}}{4}\right)c}{d}\right.$$

$$\left.-\frac{\left(\frac{2^{1/3}}{4}+\frac{I\sqrt{3}\,2^{1/3}}{4}\right)c}{d}\right)$$

$$\sqrt{\frac{x-\frac{\left(\frac{2^{1/3}}{4}+\frac{I\sqrt{3}\,2^{1/3}}{4}\right)c}{d}}{\frac{\left(\frac{2^{1/3}}{4}-\frac{I\sqrt{3}\,2^{1/3}}{4}\right)c}{d}-\frac{\left(\frac{2^{1/3}}{4}+\frac{I\sqrt{3}\,2^{1/3}}{4}\right)c}{d}}}\sqrt{\frac{x+\frac{2^{1/3}c}{2d}}{\frac{\left(\frac{2^{1/3}}{4}+\frac{I\sqrt{3}\,2^{1/3}}{4}\right)c}{d}+\frac{2^{1/3}c}{2d}}}$$

$$\sqrt{\frac{x-\frac{\left(\frac{2^{1/3}}{4}-\frac{I\sqrt{3}\,2^{1/3}}{4}\right)c}{d}}{\frac{\left(\frac{2^{1/3}}{4}+\frac{I\sqrt{3}\,2^{1/3}}{4}\right)c}{d}-\frac{\left(\frac{2^{1/3}}{4}-\frac{I\sqrt{3}\,2^{1/3}}{4}\right)c}{d}}}\ \mathrm{EllipticPi}\left(\sqrt{\frac{x-\frac{\left(\frac{2^{1/3}}{4}+\frac{I\sqrt{3}\,2^{1/3}}{4}\right)c}{d}}{\frac{\left(\frac{2^{1/3}}{4}-\frac{I\sqrt{3}\,2^{1/3}}{4}\right)c}{d}-\frac{\left(\frac{2^{1/3}}{4}+\frac{I\sqrt{3}\,2^{1/3}}{4}\right)c}{d}}},\right.$$

$$\left.\frac{\frac{\left(\frac{2^{1/3}}{4}+\frac{I\sqrt{3}\,2^{1/3}}{4}\right)c}{d}-\frac{\left(\frac{2^{1/3}}{4}-\frac{I\sqrt{3}\,2^{1/3}}{4}\right)c}{d}}{\frac{\left(\frac{2^{1/3}}{4}+\frac{I\sqrt{3}\,2^{1/3}}{4}\right)c}{d}+\frac{c}{d}},\sqrt{\frac{\frac{\left(\frac{2^{1/3}}{4}+\frac{I\sqrt{3}\,2^{1/3}}{4}\right)c}{d}-\frac{\left(\frac{2^{1/3}}{4}-\frac{I\sqrt{3}\,2^{1/3}}{4}\right)c}{d}}{\frac{\left(\frac{2^{1/3}}{4}+\frac{I\sqrt{3}\,2^{1/3}}{4}\right)c}{d}+\frac{2^{1/3}c}{2d}}}\right)\Bigg)$$

Problem 21: Result more than twice size of optimal antiderivative.

$$\int \frac{x}{(2^{2/3}+x)\sqrt{x^3+1}}\,dx$$

Optimal(type 4, 114 leaves, 4 steps):

$$-\frac{2\,2^{2/3}\arctan\left(\frac{(1+2^{1/3}x)\sqrt{3}}{\sqrt{x^3+1}}\right)\sqrt{3}}{9}+\frac{2\,(1+x)\,\mathrm{EllipticF}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}},\mathrm{I}\sqrt{3}+2\,\mathrm{I}\right)\left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)\sqrt{\frac{x^2-x+1}{(1+x+\sqrt{3})^2}}\,3^{3/4}}{9\sqrt{x^3+1}\sqrt{\frac{1+x}{(1+x+\sqrt{3})^2}}}$$

Result(type 4, 257 leaves):

$$\frac{2\left(\frac{3}{2}-\frac{\mathrm{I}\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{\mathrm{I}\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{\mathrm{I}\sqrt{3}}{2}}{-\frac{3}{2}-\frac{\mathrm{I}\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{\mathrm{I}\sqrt{3}}{2}}{-\frac{3}{2}+\frac{\mathrm{I}\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{\mathrm{I}\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{\mathrm{I}\sqrt{3}}{2}}{-\frac{3}{2}-\frac{\mathrm{I}\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}$$

$$-\frac{1}{\sqrt{x^3+1}\,(2^{2/3}-1)}\left(2\,2^{2/3}\left(\frac{3}{2}-\frac{\mathrm{I}\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{\mathrm{I}\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{\mathrm{I}\sqrt{3}}{2}}{-\frac{3}{2}-\frac{\mathrm{I}\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{\mathrm{I}\sqrt{3}}{2}}{-\frac{3}{2}+\frac{\mathrm{I}\sqrt{3}}{2}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{\mathrm{I}\sqrt{3}}{2}}},\right.\right.$$

$$\left.\left.\frac{-\frac{3}{2}+\frac{\mathrm{I}\sqrt{3}}{2}}{2^{2/3}-1},\sqrt{\frac{-\frac{3}{2}+\frac{\mathrm{I}\sqrt{3}}{2}}{-\frac{3}{2}-\frac{\mathrm{I}\sqrt{3}}{2}}}\right)\right)$$

Problem 23: Unable to integrate problem.

$$\int \frac{x}{(2^{2/3}a^{1/3}-b^{1/3}x)\sqrt{-b\,x^3+a}}\,dx$$

Optimal(type 4, 206 leaves, 4 steps):

$$-\frac{2\,2^{2/3}\arctan\left(\frac{a^{1/6}(a^{1/3}-2^{1/3}b^{1/3}x)\sqrt{3}}{\sqrt{-b\,x^3+a}}\right)\sqrt{3}}{9\,a^{1/6}\,b^{2/3}}$$

$$+\frac{2\left(a^{1/3}-b^{1/3}x\right)\operatorname{EllipticF}\left(\frac{-b^{1/3}x+a^{1/3}\left(1-\sqrt{3}\right)}{-b^{1/3}x+a^{1/3}\left(1+\sqrt{3}\right)}, I\sqrt{3}+2I\right)\left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)\sqrt{\frac{a^{2/3}+a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(-b^{1/3}x+a^{1/3}\left(1+\sqrt{3}\right)\right)^2}}\,3^{3/4}}{9\,b^{2/3}\sqrt{-b\,x^3+a}\sqrt{\frac{a^{1/3}\left(a^{1/3}-b^{1/3}x\right)}{\left(-b^{1/3}x+a^{1/3}\left(1+\sqrt{3}\right)\right)^2}}}$$

Result(type 8, 30 leaves):

$$\int \frac{x}{\left(2^{2/3}a^{1/3}-b^{1/3}x\right)\sqrt{-b\,x^3+a}}\,dx$$

Problem 24: Unable to integrate problem.

$$\int \frac{a^{1/3}-b^{1/3}x}{\left(2\,a^{1/3}+b^{1/3}x\right)\sqrt{b\,x^3-a}}\,dx$$

Optimal(type 3, 37 leaves, 2 steps):

$$-\frac{2\arctan\left(\frac{\left(a^{1/3}-b^{1/3}x\right)^2}{3\,a^{1/6}\sqrt{b\,x^3-a}}\right)}{3\,a^{1/6}b^{1/3}}$$

Result(type 8, 37 leaves):

$$\int \frac{a^{1/3}-b^{1/3}x}{\left(2\,a^{1/3}+b^{1/3}x\right)\sqrt{b\,x^3-a}}\,dx$$

Problem 25: Unable to integrate problem.

$$\int \frac{a^{1/3}+b^{1/3}x}{\left(2\,a^{1/3}-b^{1/3}x\right)\sqrt{-b\,x^3-a}}\,dx$$

Optimal(type 3, 37 leaves, 2 steps):

$$\frac{2\arctan\left(\frac{\left(a^{1/3}+b^{1/3}x\right)^2}{3\,a^{1/6}\sqrt{-b\,x^3-a}}\right)}{3\,a^{1/6}b^{1/3}}$$

Result(type 8, 38 leaves):

$$\int \frac{a^{1/3}+b^{1/3}x}{\left(2\,a^{1/3}-b^{1/3}x\right)\sqrt{-b\,x^3-a}}\,dx$$

Problem 26: Result more than twice size of optimal antiderivative.

$$\int \frac{fx+e}{(dx+c)\sqrt{-8d^3x^3+c^3}}\,dx$$

Optimal(type 4, 192 leaves, 4 steps):

$$-\frac{2(-cf+de)\operatorname{arctanh}\left(\frac{(-2dx+c)^2}{3\sqrt{c}\sqrt{-8d^3x^3+c^3}}\right)}{9c^{3/2}d^2}$$
$$-\frac{(cf+2de)(-2dx+c)\operatorname{EllipticF}\left(\frac{-2dx+c(1-\sqrt{3})}{-2dx+c(1+\sqrt{3})}, I\sqrt{3}+2I\right)\left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)\sqrt{\frac{4d^2x^2+2cdx+c^2}{(-2dx+c(1+\sqrt{3}))^2}}\,3^{3/4}}{9cd^2\sqrt{-8d^3x^3+c^3}\sqrt{\frac{c(-2dx+c)}{(-2dx+c(1+\sqrt{3}))^2}}}$$

Result(type 4, 660 leaves):

$$\frac{1}{d\sqrt{-8d^3x^3+c^3}}\left(2f\left(\frac{\left(-\frac{1}{2}+\frac{I\sqrt{3}}{2}\right)c}{2d}-\frac{\left(-\frac{1}{2}-\frac{I\sqrt{3}}{2}\right)c}{2d}\right)\right.$$
$$\sqrt{\frac{x-\frac{\left(-\frac{1}{2}-\frac{I\sqrt{3}}{2}\right)c}{2d}}{\frac{\left(-\frac{1}{2}+\frac{I\sqrt{3}}{2}\right)c}{2d}-\frac{\left(-\frac{1}{2}-\frac{I\sqrt{3}}{2}\right)c}{2d}}}\sqrt{\frac{x-\frac{c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{I\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}}}\sqrt{\frac{x-\frac{\left(-\frac{1}{2}+\frac{I\sqrt{3}}{2}\right)c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{I\sqrt{3}}{2}\right)c}{2d}-\frac{\left(-\frac{1}{2}+\frac{I\sqrt{3}}{2}\right)c}{2d}}}\operatorname{EllipticF}\left(\right.$$
$$\left.\left.\sqrt{\frac{x-\frac{\left(-\frac{1}{2}-\frac{I\sqrt{3}}{2}\right)c}{2d}}{\frac{\left(-\frac{1}{2}+\frac{I\sqrt{3}}{2}\right)c}{2d}-\frac{\left(-\frac{1}{2}-\frac{I\sqrt{3}}{2}\right)c}{2d}}}, \sqrt{\frac{\frac{\left(-\frac{1}{2}-\frac{I\sqrt{3}}{2}\right)c}{2d}-\frac{\left(-\frac{1}{2}+\frac{I\sqrt{3}}{2}\right)c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{I\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}}}\right)\right)$$

$$+\frac{1}{d^2\sqrt{-8d^3x^3+c^3}\left(\frac{\left(-\frac{1}{2}-\frac{I\sqrt{3}}{2}\right)c}{2d}+\frac{c}{d}\right)}\left(2\left(-cf+de\right)\left(\frac{\left(-\frac{1}{2}+\frac{I\sqrt{3}}{2}\right)c}{2d}\right.\right.$$

$$\left.-\frac{\left(-\frac{1}{2}-\frac{I\sqrt{3}}{2}\right)c}{2d}\right)$$

$$\sqrt{\frac{x-\frac{\left(-\frac{1}{2}-\frac{I\sqrt{3}}{2}\right)c}{2d}}{\frac{\left(-\frac{1}{2}+\frac{I\sqrt{3}}{2}\right)c}{2d}-\frac{\left(-\frac{1}{2}-\frac{I\sqrt{3}}{2}\right)c}{2d}}}\sqrt{\frac{x-\frac{c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{I\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}}}\sqrt{\frac{x-\frac{\left(-\frac{1}{2}+\frac{I\sqrt{3}}{2}\right)c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{I\sqrt{3}}{2}\right)c}{2d}-\frac{\left(-\frac{1}{2}+\frac{I\sqrt{3}}{2}\right)c}{2d}}}\,\mathrm{EllipticPi}\left(\right.$$

$$\left.\left.\sqrt{\frac{x-\frac{\left(-\frac{1}{2}-\frac{I\sqrt{3}}{2}\right)c}{2d}}{\frac{\left(-\frac{1}{2}+\frac{I\sqrt{3}}{2}\right)c}{2d}-\frac{\left(-\frac{1}{2}-\frac{I\sqrt{3}}{2}\right)c}{2d}}},\frac{\frac{\left(-\frac{1}{2}-\frac{I\sqrt{3}}{2}\right)c}{2d}-\frac{\left(-\frac{1}{2}+\frac{I\sqrt{3}}{2}\right)c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{I\sqrt{3}}{2}\right)c}{2d}+\frac{c}{d}},\sqrt{\frac{\frac{\left(-\frac{1}{2}-\frac{I\sqrt{3}}{2}\right)c}{2d}-\frac{\left(-\frac{1}{2}+\frac{I\sqrt{3}}{2}\right)c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{I\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}}}\right)\right)$$

Problem 27: Result more than twice size of optimal antiderivative.

$$\int \frac{x}{(2-x)\sqrt{x^3+1}}\,dx$$

Optimal(type 4, 104 leaves, 4 steps):

$$\frac{4\,\mathrm{arctanh}\left(\frac{(1+x)^2}{3\sqrt{x^3+1}}\right)}{9}-\frac{2(1+x)\,\mathrm{EllipticF}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}},I\sqrt{3}+2I\right)\left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)\sqrt{\frac{x^2-x+1}{\left(1+x+\sqrt{3}\right)^2}}\,3^{3/4}}{9\sqrt{x^3+1}\sqrt{\frac{1+x}{\left(1+x+\sqrt{3}\right)^2}}}$$

Result(type 4, 239 leaves):

$$-\frac{2\left(\frac{3}{2}-\frac{I\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{I\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{I\sqrt{3}}{2}}{-\frac{3}{2}-\frac{I\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{I\sqrt{3}}{2}}{-\frac{3}{2}+\frac{I\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{I\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{I\sqrt{3}}{2}}{-\frac{3}{2}-\frac{I\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}$$

$$+\frac{4\left(\frac{3}{2}-\frac{I\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{I\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{I\sqrt{3}}{2}}{-\frac{3}{2}-\frac{I\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{I\sqrt{3}}{2}}{-\frac{3}{2}+\frac{I\sqrt{3}}{2}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{I\sqrt{3}}{2}}},\frac{1}{2}-\frac{I\sqrt{3}}{6},\sqrt{\frac{-\frac{3}{2}+\frac{I\sqrt{3}}{2}}{-\frac{3}{2}-\frac{I\sqrt{3}}{2}}}\right)}{3\sqrt{x^3+1}}$$

Problem 28: Result more than twice size of optimal antiderivative.

$$\int \frac{x}{(d\,x+c)\sqrt{-8\,d^3\,x^3+c^3}}\,dx$$

Optimal(type 4, 173 leaves, 4 steps):

$$\frac{2\,\mathrm{arctanh}\left(\frac{(-2\,d\,x+c)^2}{3\sqrt{c}\,\sqrt{-8\,d^3\,x^3+c^3}}\right)}{9\,d^2\sqrt{c}}-\frac{(-2\,d\,x+c)\,\mathrm{EllipticF}\left(\frac{-2\,d\,x+c\,(1-\sqrt{3})}{-2\,d\,x+c\,(1+\sqrt{3})},I\sqrt{3}+2\,I\right)\left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)\sqrt{\frac{4\,d^2\,x^2+2\,c\,d\,x+c^2}{\left(-2\,d\,x+c\,(1+\sqrt{3})\right)^2}}\,3^{3/4}}{9\,d^2\sqrt{-8\,d^3\,x^3+c^3}\,\sqrt{\frac{c\,(-2\,d\,x+c)}{\left(-2\,d\,x+c\,(1+\sqrt{3})\right)^2}}}$$

Result(type 4, 652 leaves):

$$\frac{1}{d\sqrt{-8\,d^3\,x^3+c^3}}\left(2\left(\frac{\left(-\frac{1}{2}+\frac{I\sqrt{3}}{2}\right)c}{2\,d}\right.\right.$$

$$\left.-\frac{\left(-\frac{1}{2}-\frac{I\sqrt{3}}{2}\right)c}{2\,d}\right)$$

$$\sqrt{\frac{x-\frac{\left(-\frac{1}{2}-\frac{I\sqrt{3}}{2}\right)c}{2d}}{\frac{\left(-\frac{1}{2}+\frac{I\sqrt{3}}{2}\right)c}{2d}-\frac{\left(-\frac{1}{2}-\frac{I\sqrt{3}}{2}\right)c}{2d}}}\sqrt{\frac{x-\frac{c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{I\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}}}\sqrt{\frac{x-\frac{\left(-\frac{1}{2}+\frac{I\sqrt{3}}{2}\right)c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{I\sqrt{3}}{2}\right)c}{2d}-\frac{\left(-\frac{1}{2}+\frac{I\sqrt{3}}{2}\right)c}{2d}}}\,\mathrm{EllipticF}\left(\vphantom{\frac{\frac{1}{2}}{\frac{1}{2}}}\right.$$

$$\left.\sqrt{\frac{x-\frac{\left(-\frac{1}{2}-\frac{I\sqrt{3}}{2}\right)c}{2d}}{\frac{\left(-\frac{1}{2}+\frac{I\sqrt{3}}{2}\right)c}{2d}-\frac{\left(-\frac{1}{2}-\frac{I\sqrt{3}}{2}\right)c}{2d}}},\sqrt{\frac{\frac{\left(-\frac{1}{2}-\frac{I\sqrt{3}}{2}\right)c}{2d}-\frac{\left(-\frac{1}{2}+\frac{I\sqrt{3}}{2}\right)c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{I\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}}}\right)\Bigg)$$

$$-\frac{1}{d^2\sqrt{-8d^3x^3+c^3}\left(\frac{\left(-\frac{1}{2}-\frac{I\sqrt{3}}{2}\right)c}{2d}+\frac{c}{d}\right)}\left(2c\left(\frac{\left(-\frac{1}{2}+\frac{I\sqrt{3}}{2}\right)c}{2d}\right.\right.$$

$$\left.-\frac{\left(-\frac{1}{2}-\frac{I\sqrt{3}}{2}\right)c}{2d}\right)$$

$$\sqrt{\frac{x-\frac{\left(-\frac{1}{2}-\frac{I\sqrt{3}}{2}\right)c}{2d}}{\frac{\left(-\frac{1}{2}+\frac{I\sqrt{3}}{2}\right)c}{2d}-\frac{\left(-\frac{1}{2}-\frac{I\sqrt{3}}{2}\right)c}{2d}}}\sqrt{\frac{x-\frac{c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{I\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}}}\sqrt{\frac{x-\frac{\left(-\frac{1}{2}+\frac{I\sqrt{3}}{2}\right)c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{I\sqrt{3}}{2}\right)c}{2d}-\frac{\left(-\frac{1}{2}+\frac{I\sqrt{3}}{2}\right)c}{2d}}}\,\mathrm{EllipticPi}\left(\vphantom{\frac{\frac{1}{2}}{\frac{1}{2}}}\right.$$

$$\left.\sqrt{\frac{x-\frac{\left(-\frac{1}{2}-\frac{I\sqrt{3}}{2}\right)c}{2d}}{\frac{\left(-\frac{1}{2}+\frac{I\sqrt{3}}{2}\right)c}{2d}-\frac{\left(-\frac{1}{2}-\frac{I\sqrt{3}}{2}\right)c}{2d}}},\frac{\frac{\left(-\frac{1}{2}-\frac{I\sqrt{3}}{2}\right)c}{2d}-\frac{\left(-\frac{1}{2}+\frac{I\sqrt{3}}{2}\right)c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{I\sqrt{3}}{2}\right)c}{2d}+\frac{c}{d}},\sqrt{\frac{\frac{\left(-\frac{1}{2}-\frac{I\sqrt{3}}{2}\right)c}{2d}-\frac{\left(-\frac{1}{2}+\frac{I\sqrt{3}}{2}\right)c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{I\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}}}\right)\Bigg)$$

Problem 29: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1+x+\sqrt{3}}{\left(1+x-\sqrt{3}\right)\sqrt{x^3+1}}\, dx$$

Optimal(type 3, 32 leaves, 2 steps):

$$-\frac{2\,\operatorname{arctanh}\left(\frac{(1+x)\sqrt{-3+2\sqrt{3}}}{\sqrt{x^3+1}}\right)}{\sqrt{-3+2\sqrt{3}}}$$

Result(type 4, 244 leaves):

$$\frac{2\left(\frac{3}{2}-\frac{I\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{I\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{I\sqrt{3}}{2}}{-\frac{3}{2}-\frac{I\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{I\sqrt{3}}{2}}{-\frac{3}{2}+\frac{I\sqrt{3}}{2}}}\operatorname{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{I\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{I\sqrt{3}}{2}}{-\frac{3}{2}-\frac{I\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}$$

$$-\frac{1}{\sqrt{x^3+1}}\left(4\left(\frac{3}{2}-\frac{I\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{I\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{I\sqrt{3}}{2}}{-\frac{3}{2}-\frac{I\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{I\sqrt{3}}{2}}{-\frac{3}{2}+\frac{I\sqrt{3}}{2}}}\operatorname{EllipticPi}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{I\sqrt{3}}{2}}},\right.\right.$$

$$\left.\left.-\frac{\left(-\frac{3}{2}+\frac{I\sqrt{3}}{2}\right)\sqrt{3}}{3},\sqrt{\frac{-\frac{3}{2}+\frac{I\sqrt{3}}{2}}{-\frac{3}{2}-\frac{I\sqrt{3}}{2}}}\right)\right)$$

Problem 30: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1-x+\sqrt{3}}{\left(1-x-\sqrt{3}\right)\sqrt{-x^3+1}}\, dx$$

Optimal(type 3, 36 leaves, 2 steps):

$$\frac{2\,\operatorname{arctanh}\left(\frac{(1-x)\sqrt{-3+2\sqrt{3}}}{\sqrt{-x^3+1}}\right)}{\sqrt{-3+2\sqrt{3}}}$$

Result(type 4, 242 leaves):

$$-\frac{2 I\sqrt{3}\sqrt{I\left(x+\frac{1}{2}-\frac{I\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{I\sqrt{3}}{2}}}\sqrt{-I\left(x+\frac{1}{2}+\frac{I\sqrt{3}}{2}\right)\sqrt{3}}\ \mathrm{EllipticF}\left(\frac{\sqrt{3}\sqrt{I\left(x+\frac{1}{2}-\frac{I\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{I\sqrt{3}}{-\frac{3}{2}+\frac{I\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}}$$

$$+\frac{1}{\sqrt{-x^3+1}\left(-\frac{3}{2}+\frac{I\sqrt{3}}{2}+\sqrt{3}\right)}\left(4 I\sqrt{I\left(x+\frac{1}{2}-\frac{I\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{I\sqrt{3}}{2}}}\sqrt{-I\left(x+\frac{1}{2}+\frac{I\sqrt{3}}{2}\right)\sqrt{3}}\ \mathrm{EllipticPi}\left(\frac{1}{3}\left(\sqrt{3}\right.\right.\right.$$
$$\left.\left.\left.\sqrt{I\left(x+\frac{1}{2}-\frac{I\sqrt{3}}{2}\right)\sqrt{3}}\right),\frac{I\sqrt{3}}{-\frac{3}{2}+\frac{I\sqrt{3}}{2}+\sqrt{3}},\sqrt{\frac{I\sqrt{3}}{-\frac{3}{2}+\frac{I\sqrt{3}}{2}}}\right)\right)$$

Problem 31: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\frac{1+x-\sqrt{3}}{\left(1+x+\sqrt{3}\right)\sqrt{x^3+1}}\,dx$$

Optimal(type 3, 32 leaves, 2 steps):

$$-\frac{2\arctan\left(\frac{(1+x)\sqrt{3+2\sqrt{3}}}{\sqrt{x^3+1}}\right)}{\sqrt{3+2\sqrt{3}}}$$

Result(type 4, 244 leaves):

$$\frac{2\left(\frac{3}{2}-\frac{I\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{I\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{I\sqrt{3}}{2}}{-\frac{3}{2}-\frac{I\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{I\sqrt{3}}{2}}{-\frac{3}{2}+\frac{I\sqrt{3}}{2}}}\ \mathrm{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{I\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{I\sqrt{3}}{2}}{-\frac{3}{2}-\frac{I\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}$$

$$-\frac{1}{\sqrt{x^3+1}}\left(4\left(\frac{3}{2}-\frac{I\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{I\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{I\sqrt{3}}{2}}{-\frac{3}{2}-\frac{I\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{I\sqrt{3}}{2}}{-\frac{3}{2}+\frac{I\sqrt{3}}{2}}}\ \mathrm{EllipticPi}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{I\sqrt{3}}{2}}},\right.\right.$$

$$\frac{\left(-\frac{3}{2}+\frac{I\sqrt{3}}{2}\right)\sqrt{3}}{3},\ \sqrt{\frac{-\frac{3}{2}+\frac{I\sqrt{3}}{2}}{-\frac{3}{2}-\frac{I\sqrt{3}}{2}}}\ \Bigg)\Bigg)$$

Problem 32: Unable to integrate problem.

$$\int \frac{b^{1/3}x+a^{1/3}\left(1-\sqrt{3}\right)}{\left(b^{1/3}x+a^{1/3}\left(1+\sqrt{3}\right)\right)\sqrt{b\,x^3+a}}\,dx$$

Optimal(type 3, 49 leaves, 2 steps):

$$-\frac{2\arctan\left(\frac{a^{1/6}\left(a^{1/3}+b^{1/3}x\right)\sqrt{3+2\sqrt{3}}}{\sqrt{b\,x^3+a}}\right)}{a^{1/6}b^{1/3}\sqrt{3+2\sqrt{3}}}$$

Result(type 8, 46 leaves):

$$\int \frac{b^{1/3}x+a^{1/3}\left(1-\sqrt{3}\right)}{\left(b^{1/3}x+a^{1/3}\left(1+\sqrt{3}\right)\right)\sqrt{b\,x^3+a}}\,dx$$

Problem 33: Unable to integrate problem.

$$\int \frac{-b^{1/3}x+a^{1/3}\left(1-\sqrt{3}\right)}{\left(-b^{1/3}x+a^{1/3}\left(1+\sqrt{3}\right)\right)\sqrt{-b\,x^3+a}}\,dx$$

Optimal(type 3, 51 leaves, 2 steps):

$$\frac{2\arctan\left(\frac{a^{1/6}\left(a^{1/3}-b^{1/3}x\right)\sqrt{3+2\sqrt{3}}}{\sqrt{-b\,x^3+a}}\right)}{a^{1/6}b^{1/3}\sqrt{3+2\sqrt{3}}}$$

Result(type 8, 49 leaves):

$$\int \frac{-b^{1/3}x+a^{1/3}\left(1-\sqrt{3}\right)}{\left(-b^{1/3}x+a^{1/3}\left(1+\sqrt{3}\right)\right)\sqrt{-b\,x^3+a}}\,dx$$

Problem 34: Unable to integrate problem.

$$\int \frac{-b^{1/3}x+a^{1/3}\left(1-\sqrt{3}\right)}{\left(-b^{1/3}x+a^{1/3}\left(1+\sqrt{3}\right)\right)\sqrt{b\,x^3-a}}\,dx$$

Optimal(type 3, 52 leaves, 2 steps):

$$\frac{2 \operatorname{arctanh}\left(\frac{a^{1/6} \left(a^{1/3} - b^{1/3} x\right) \sqrt{3 + 2\sqrt{3}}}{\sqrt{b x^3 - a}}\right)}{a^{1/6} b^{1/3} \sqrt{3 + 2\sqrt{3}}}$$

Result(type 8, 50 leaves):

$$\int \frac{-b^{1/3} x + a^{1/3} \left(1 - \sqrt{3}\right)}{\left(-b^{1/3} x + a^{1/3} \left(1 + \sqrt{3}\right)\right) \sqrt{b x^3 - a}}\, dx$$

Problem 35: Unable to integrate problem.

$$\int \frac{b^{1/3} x + a^{1/3} \left(1 - \sqrt{3}\right)}{\left(b^{1/3} x + a^{1/3} \left(1 + \sqrt{3}\right)\right) \sqrt{-b x^3 - a}}\, dx$$

Optimal(type 3, 52 leaves, 2 steps):

$$-\frac{2 \operatorname{arctanh}\left(\frac{a^{1/6} \left(a^{1/3} + b^{1/3} x\right) \sqrt{3 + 2\sqrt{3}}}{\sqrt{-b x^3 - a}}\right)}{a^{1/6} b^{1/3} \sqrt{3 + 2\sqrt{3}}}$$

Result(type 8, 49 leaves):

$$\int \frac{b^{1/3} x + a^{1/3} \left(1 - \sqrt{3}\right)}{\left(b^{1/3} x + a^{1/3} \left(1 + \sqrt{3}\right)\right) \sqrt{-b x^3 - a}}\, dx$$

Problem 36: Unable to integrate problem.

$$\int \frac{1 - \left(\frac{b}{a}\right)^{1/3} x - \sqrt{3}}{\left(1 - \left(\frac{b}{a}\right)^{1/3} x + \sqrt{3}\right) \sqrt{-b x^3 + a}}\, dx$$

Optimal(type 3, 57 leaves, 2 steps):

$$\frac{2 \arctan\left(\frac{\left(1 - \left(\frac{b}{a}\right)^{1/3} x\right) \sqrt{a} \sqrt{3 + 2\sqrt{3}}}{\sqrt{-b x^3 + a}}\right)}{\left(\frac{b}{a}\right)^{1/3} \sqrt{a} \sqrt{3 + 2\sqrt{3}}}$$

Result(type 8, 47 leaves):

$$\int \frac{1-\left(\frac{b}{a}\right)^{1/3} x-\sqrt{3}}{\left(1-\left(\frac{b}{a}\right)^{1/3} x+\sqrt{3}\right)\sqrt{-b x^3+a}}\, dx$$

Problem 37: Unable to integrate problem.

$$\int \frac{1+\left(\frac{b}{a}\right)^{1/3} x-\sqrt{3}}{\left(1+\left(\frac{b}{a}\right)^{1/3} x+\sqrt{3}\right)\sqrt{-b x^3-a}}\, dx$$

Optimal(type 3, 58 leaves, 2 steps):

$$-\frac{2 \operatorname{arctanh}\left(\frac{\left(1+\left(\frac{b}{a}\right)^{1/3} x\right)\sqrt{a}\sqrt{3+2\sqrt{3}}}{\sqrt{-b x^3-a}}\right)}{\left(\frac{b}{a}\right)^{1/3}\sqrt{a}\sqrt{3+2\sqrt{3}}}$$

Result(type 8, 47 leaves):

$$\int \frac{1+\left(\frac{b}{a}\right)^{1/3} x-\sqrt{3}}{\left(1+\left(\frac{b}{a}\right)^{1/3} x+\sqrt{3}\right)\sqrt{-b x^3-a}}\, dx$$

Problem 38: Result more than twice size of optimal antiderivative.

$$\int \frac{1+x}{\left(1+x-\sqrt{3}\right)\sqrt{x^3+1}}\, dx$$

Optimal(type 4, 119 leaves, 4 steps):

$$\frac{(1+x)\operatorname{EllipticF}\left(\frac{1+x-\sqrt{3}}{1+x+\sqrt{3}}, \mathrm{I}\sqrt{3}+2\,\mathrm{I}\right)\left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)\sqrt{\frac{x^2-x+1}{\left(1+x+\sqrt{3}\right)^2}}\, 3^{3/4}}{3\sqrt{x^3+1}\sqrt{\frac{1+x}{\left(1+x+\sqrt{3}\right)^2}}}-\frac{\operatorname{arctanh}\left(\frac{(1+x)\sqrt{-3+2\sqrt{3}}}{\sqrt{x^3+1}}\right)}{\sqrt{-3+2\sqrt{3}}}$$

Result(type 4, 244 leaves):

$$\frac{2\left(\frac{3}{2}-\frac{I\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{I\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{I\sqrt{3}}{2}}{-\frac{3}{2}-\frac{I\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{I\sqrt{3}}{2}}{-\frac{3}{2}+\frac{I\sqrt{3}}{2}}}\operatorname{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{I\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{I\sqrt{3}}{2}}{-\frac{3}{2}-\frac{I\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}$$

$$-\frac{1}{\sqrt{x^3+1}}\left(2\left(\frac{3}{2}-\frac{I\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{I\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{I\sqrt{3}}{2}}{-\frac{3}{2}-\frac{I\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{I\sqrt{3}}{2}}{-\frac{3}{2}+\frac{I\sqrt{3}}{2}}}\operatorname{EllipticPi}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{I\sqrt{3}}{2}}},\right.\right.$$

$$\left.\left.-\frac{\left(-\frac{3}{2}+\frac{I\sqrt{3}}{2}\right)\sqrt{3}}{3},\sqrt{\frac{-\frac{3}{2}+\frac{I\sqrt{3}}{2}}{-\frac{3}{2}-\frac{I\sqrt{3}}{2}}}\right)\right)$$

Problem 39: Unable to integrate problem.

$$\int \frac{fx+e}{\left(b^{1/3}x+a^{1/3}\left(1-\sqrt{3}\right)\right)\sqrt{b x^3+a}}\,dx$$

Optimal(type 4, 243 leaves, 4 steps):

$$-\frac{\operatorname{arctanh}\left(\frac{a^{1/6}\left(a^{1/3}+b^{1/3}x\right)\sqrt{-3+2\sqrt{3}}}{\sqrt{b x^3+a}}\right)\left(b^{1/3}e-a^{1/3}f\left(1-\sqrt{3}\right)\right)}{b^{2/3}\sqrt{a}\sqrt{-9+6\sqrt{3}}}$$

$$-\frac{1}{3a^{1/3}b^{2/3}\sqrt{b x^3+a}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(b^{1/3}x+a^{1/3}\left(1+\sqrt{3}\right)\right)^2}}}\left(\left(a^{1/3}+b^{1/3}x\right)\operatorname{EllipticF}\left(\frac{b^{1/3}x+a^{1/3}\left(1-\sqrt{3}\right)}{b^{1/3}x+a^{1/3}\left(1+\sqrt{3}\right)},I\sqrt{3}+2I\right)\left(b^{1/3}e\right.\right.$$

$$\left.\left.-a^{1/3}f\left(1+\sqrt{3}\right)\right)\left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(b^{1/3}x+a^{1/3}\left(1+\sqrt{3}\right)\right)^2}}\,3^{1/4}\right)$$

Result(type 8, 36 leaves):

$$\int \frac{fx+e}{\left(b^{1/3}x+a^{1/3}\left(1-\sqrt{3}\right)\right)\sqrt{b x^3+a}}\,dx$$

Problem 40: Unable to integrate problem.

$$\int \frac{fx+e}{\left(-b^{1/3}x+a^{1/3}\left(1-\sqrt{3}\right)\right)\sqrt{b\,x^3-a}}\,dx$$

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Optimal(type 4, 255 leaves, 4 steps):
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$$\frac{1}{3\,a^{1/3}b^{2/3}\sqrt{b\,x^3-a}\,\sqrt{-\frac{a^{1/3}\,(a^{1/3}-b^{1/3}x)}{\left(-b^{1/3}x+a^{1/3}\left(1-\sqrt{3}\right)\right)^2}}}\left((a^{1/3}-b^{1/3}x)\,\mathrm{EllipticF}\left(\frac{-b^{1/3}x+a^{1/3}\left(1+\sqrt{3}\right)}{-b^{1/3}x+a^{1/3}\left(1-\sqrt{3}\right)},2\,\mathrm{I}-\mathrm{I}\sqrt{3}\right)\left(b^{1/3}e+a^{1/3}f\left(1\right.\right.\right.$$

$$\left.\left.\left.+\sqrt{3}\right)\right)\sqrt{\frac{a^{2/3}+a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(-b^{1/3}x+a^{1/3}\left(1-\sqrt{3}\right)\right)^2}}\left(\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)3^{1/4}\right)+\frac{\arctan\left(\frac{a^{1/6}\,(a^{1/3}-b^{1/3}x)\sqrt{-3+2\sqrt{3}}}{\sqrt{b\,x^3-a}}\right)\left(b^{1/3}e+a^{1/3}f\left(1-\sqrt{3}\right)\right)}{b^{2/3}\sqrt{a}\,\sqrt{-9+6\sqrt{3}}}$$

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Result(type 8, 39 leaves):
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$$\int \frac{fx+e}{\left(-b^{1/3}x+a^{1/3}\left(1-\sqrt{3}\right)\right)\sqrt{b\,x^3-a}}\,dx$$

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Problem 41: Unable to integrate problem.
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$$\int \frac{x}{\left(-b^{1/3}x+a^{1/3}\left(1-\sqrt{3}\right)\right)\sqrt{b\,x^3-a}}\,dx$$

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Optimal(type 4, 208 leaves, 4 steps):
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$$-\frac{\arctan\left(\frac{a^{1/6}\,(a^{1/3}-b^{1/3}x)\sqrt{-3+2\sqrt{3}}}{\sqrt{b\,x^3-a}}\right)\sqrt{2}\,3^{1/4}}{3\,a^{1/6}b^{2/3}}$$

$$+\frac{(a^{1/3}-b^{1/3}x)\,\mathrm{EllipticF}\left(\frac{-b^{1/3}x+a^{1/3}\left(1+\sqrt{3}\right)}{-b^{1/3}x+a^{1/3}\left(1-\sqrt{3}\right)},2\,\mathrm{I}-\mathrm{I}\sqrt{3}\right)\sqrt{2}\sqrt{\frac{a^{2/3}+a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(-b^{1/3}x+a^{1/3}\left(1-\sqrt{3}\right)\right)^2}}\;3^{1/4}}{3\,b^{2/3}\sqrt{b\,x^3-a}\,\sqrt{-\frac{a^{1/3}\,(a^{1/3}-b^{1/3}x)}{\left(-b^{1/3}x+a^{1/3}\left(1-\sqrt{3}\right)\right)^2}}}$$

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Result(type 8, 35 leaves):
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$$\int \frac{x}{\left(-b^{1/3}x+a^{1/3}\left(1-\sqrt{3}\right)\right)\sqrt{b\,x^3-a}}\,dx$$

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Problem 49: Unable to integrate problem.
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$$\int \frac{x^3 (fx+e)^n}{b x^3 + a}\, dx$$

Optimal(type 5, 245 leaves, 7 steps):

$$\frac{(fx+e)^{1+n}}{bf(1+n)} + \frac{a^{1/3} (fx+e)^{1+n} \operatorname{hypergeom}\left([1, 1+n], [n+2], \frac{b^{1/3}(fx+e)}{b^{1/3} e - a^{1/3} f}\right)}{3 b \left(b^{1/3} e - a^{1/3} f\right) (1+n)}$$
$$+ \frac{a^{1/3} (fx+e)^{1+n} \operatorname{hypergeom}\left([1, 1+n], [n+2], \frac{(-1)^{2/3} b^{1/3}(fx+e)}{(-1)^{2/3} b^{1/3} e - a^{1/3} f}\right)}{3 b \left((-1)^{2/3} b^{1/3} e - a^{1/3} f\right) (1+n)}$$
$$- \frac{a^{1/3} (fx+e)^{1+n} \operatorname{hypergeom}\left([1, 1+n], [n+2], \frac{(-1)^{1/3} b^{1/3}(fx+e)}{(-1)^{1/3} b^{1/3} e + a^{1/3} f}\right)}{3 b \left((-1)^{1/3} b^{1/3} e + a^{1/3} f\right) (1+n)}$$

Result(type 8, 22 leaves):

$$\int \frac{x^3 (fx+e)^n}{b x^3 + a}\, dx$$

Problem 50: Unable to integrate problem.

$$\int \frac{x (fx+e)^n}{b x^3 + a}\, dx$$

Optimal(type 5, 230 leaves, 5 steps):

$$\frac{(fx+e)^{1+n} \operatorname{hypergeom}\left([1, 1+n], [n+2], \frac{b^{1/3}(fx+e)}{b^{1/3} e - a^{1/3} f}\right)}{3 a^{1/3} b^{1/3} \left(b^{1/3} e - a^{1/3} f\right) (1+n)} - \frac{(-1)^{1/3} (fx+e)^{1+n} \operatorname{hypergeom}\left([1, 1+n], [n+2], \frac{(-1)^{2/3} b^{1/3}(fx+e)}{(-1)^{2/3} b^{1/3} e - a^{1/3} f}\right)}{3 a^{1/3} b^{1/3} \left((-1)^{2/3} b^{1/3} e - a^{1/3} f\right) (1+n)}$$
$$- \frac{(-1)^{2/3} (fx+e)^{1+n} \operatorname{hypergeom}\left([1, 1+n], [n+2], \frac{(-1)^{1/3} b^{1/3}(fx+e)}{(-1)^{1/3} b^{1/3} e + a^{1/3} f}\right)}{3 a^{1/3} b^{1/3} \left((-1)^{1/3} b^{1/3} e + a^{1/3} f\right) (1+n)}$$

Result(type 8, 20 leaves):

$$\int \frac{x (fx+e)^n}{b x^3 + a}\, dx$$

Problem 51: Unable to integrate problem.

$$\int \frac{(e^3 x^3 + d^3)^p}{e x + d}\, dx$$

Optimal(type 6, 123 leaves, ? steps):

$$\frac{(e^3x^3+d^3)^p \, AppellF1\left(p, -p, -p, 1+p, -\frac{2\,(ex+d)}{d\left(-3+I\sqrt{3}\right)}, \frac{2\,(ex+d)}{d\left(3+I\sqrt{3}\right)}\right)}{ep\left(1+\frac{2\,(ex+d)}{d\left(-3+I\sqrt{3}\right)}\right)^p\left(1-\frac{2\,(ex+d)}{d\left(3+I\sqrt{3}\right)}\right)^p}$$

Result(type 8, 23 leaves):

$$\int \frac{(e^3x^3+d^3)^p}{ex+d}\, dx$$

Problem 52: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{-x^2+2x+2}{(x^2+2)\sqrt{x^3-1}}\, dx$$

Optimal(type 3, 16 leaves, 2 steps):

$$-2\,\mathrm{arctanh}\left(\frac{1-x}{\sqrt{x^3-1}}\right)$$

Result(type 4, 1655 leaves):

$$-\frac{2\left(-\frac{3}{2}-\frac{I\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{I\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{I\sqrt{3}}{2}}{\frac{3}{2}-\frac{I\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{I\sqrt{3}}{2}}{\frac{3}{2}+\frac{I\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{I\sqrt{3}}{2}}}, \sqrt{\frac{\frac{3}{2}+\frac{I\sqrt{3}}{2}}{\frac{3}{2}-\frac{I\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}$$

$$-\frac{1}{\sqrt{x^3-1}\left(1-I\sqrt{2}\right)}\left(3\sqrt{\frac{x}{-\frac{3}{2}-\frac{I\sqrt{3}}{2}}-\frac{1}{-\frac{3}{2}-\frac{I\sqrt{3}}{2}}}\sqrt{\frac{x}{\frac{3}{2}-\frac{I\sqrt{3}}{2}}+\frac{1}{2\left(\frac{3}{2}-\frac{I\sqrt{3}}{2}\right)}-\frac{I\sqrt{3}}{2\left(\frac{3}{2}-\frac{I\sqrt{3}}{2}\right)}}\right.$$

$$\left.\sqrt{\frac{x}{\frac{3}{2}+\frac{I\sqrt{3}}{2}}+\frac{1}{2\left(\frac{3}{2}+\frac{I\sqrt{3}}{2}\right)}+\frac{I\sqrt{3}}{2\left(\frac{3}{2}+\frac{I\sqrt{3}}{2}\right)}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{I\sqrt{3}}{2}}}, \frac{\frac{3}{2}+\frac{I\sqrt{3}}{2}}{1-I\sqrt{2}}, \sqrt{\frac{\frac{3}{2}+\frac{I\sqrt{3}}{2}}{\frac{3}{2}-\frac{I\sqrt{3}}{2}}}\right)\right)$$

$$-\frac{1}{\sqrt{x^3-1}\left(1-I\sqrt{2}\right)}\left(I\sqrt{\frac{x}{-\frac{3}{2}-\frac{I\sqrt{3}}{2}}-\frac{1}{-\frac{3}{2}-\frac{I\sqrt{3}}{2}}}\sqrt{\frac{x}{\frac{3}{2}-\frac{I\sqrt{3}}{2}}+\frac{1}{2\left(\frac{3}{2}-\frac{I\sqrt{3}}{2}\right)}-\frac{I\sqrt{3}}{2\left(\frac{3}{2}-\frac{I\sqrt{3}}{2}\right)}}\right.$$

$$\sqrt{\frac{x}{\frac{3}{2}+\frac{\mathrm{I}\sqrt{3}}{2}}+\frac{1}{2\left(\frac{3}{2}+\frac{\mathrm{I}\sqrt{3}}{2}\right)}+\frac{\mathrm{I}\sqrt{3}}{2\left(\frac{3}{2}+\frac{\mathrm{I}\sqrt{3}}{2}\right)}}\ \mathrm{EllipticPi}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{\mathrm{I}\sqrt{3}}{2}}},\ \frac{\frac{3}{2}+\frac{\mathrm{I}\sqrt{3}}{2}}{1-\mathrm{I}\sqrt{2}},\ \sqrt{\frac{\frac{3}{2}+\frac{\mathrm{I}\sqrt{3}}{2}}{\frac{3}{2}-\frac{\mathrm{I}\sqrt{3}}{2}}}\right)\sqrt{3}\ \Bigg)$$

$$+\frac{1}{\sqrt{x^3-1}\,\left(1-\mathrm{I}\sqrt{2}\right)}\Bigg(3\,\mathrm{I}\sqrt{2}\,\sqrt{\frac{x}{-\frac{3}{2}-\frac{\mathrm{I}\sqrt{3}}{2}}-\frac{1}{-\frac{3}{2}-\frac{\mathrm{I}\sqrt{3}}{2}}}\ \sqrt{\frac{x}{\frac{3}{2}-\frac{\mathrm{I}\sqrt{3}}{2}}+\frac{1}{2\left(\frac{3}{2}-\frac{\mathrm{I}\sqrt{3}}{2}\right)}-\frac{\mathrm{I}\sqrt{3}}{2\left(\frac{3}{2}-\frac{\mathrm{I}\sqrt{3}}{2}\right)}}$$

$$\sqrt{\frac{x}{\frac{3}{2}+\frac{\mathrm{I}\sqrt{3}}{2}}+\frac{1}{2\left(\frac{3}{2}+\frac{\mathrm{I}\sqrt{3}}{2}\right)}+\frac{\mathrm{I}\sqrt{3}}{2\left(\frac{3}{2}+\frac{\mathrm{I}\sqrt{3}}{2}\right)}}\ \mathrm{EllipticPi}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{\mathrm{I}\sqrt{3}}{2}}},\ \frac{\frac{3}{2}+\frac{\mathrm{I}\sqrt{3}}{2}}{1-\mathrm{I}\sqrt{2}},\ \sqrt{\frac{\frac{3}{2}+\frac{\mathrm{I}\sqrt{3}}{2}}{\frac{3}{2}-\frac{\mathrm{I}\sqrt{3}}{2}}}\right)\Bigg)$$

$$-\frac{1}{\sqrt{x^3-1}\,\left(1-\mathrm{I}\sqrt{2}\right)}\Bigg(\sqrt{2}\,\sqrt{\frac{x}{-\frac{3}{2}-\frac{\mathrm{I}\sqrt{3}}{2}}-\frac{1}{-\frac{3}{2}-\frac{\mathrm{I}\sqrt{3}}{2}}}\ \sqrt{\frac{x}{\frac{3}{2}-\frac{\mathrm{I}\sqrt{3}}{2}}+\frac{1}{2\left(\frac{3}{2}-\frac{\mathrm{I}\sqrt{3}}{2}\right)}-\frac{\mathrm{I}\sqrt{3}}{2\left(\frac{3}{2}-\frac{\mathrm{I}\sqrt{3}}{2}\right)}}$$

$$\sqrt{\frac{x}{\frac{3}{2}+\frac{\mathrm{I}\sqrt{3}}{2}}+\frac{1}{2\left(\frac{3}{2}+\frac{\mathrm{I}\sqrt{3}}{2}\right)}+\frac{\mathrm{I}\sqrt{3}}{2\left(\frac{3}{2}+\frac{\mathrm{I}\sqrt{3}}{2}\right)}}\ \mathrm{EllipticPi}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{\mathrm{I}\sqrt{3}}{2}}},\ \frac{\frac{3}{2}+\frac{\mathrm{I}\sqrt{3}}{2}}{1-\mathrm{I}\sqrt{2}},\ \sqrt{\frac{\frac{3}{2}+\frac{\mathrm{I}\sqrt{3}}{2}}{\frac{3}{2}-\frac{\mathrm{I}\sqrt{3}}{2}}}\right)\sqrt{3}\ \Bigg)$$

$$-\frac{1}{\sqrt{x^3-1}\,\left(\mathrm{I}\sqrt{2}+1\right)}\Bigg(3\,\sqrt{\frac{x}{-\frac{3}{2}-\frac{\mathrm{I}\sqrt{3}}{2}}-\frac{1}{-\frac{3}{2}-\frac{\mathrm{I}\sqrt{3}}{2}}}\ \sqrt{\frac{x}{\frac{3}{2}-\frac{\mathrm{I}\sqrt{3}}{2}}+\frac{1}{2\left(\frac{3}{2}-\frac{\mathrm{I}\sqrt{3}}{2}\right)}-\frac{\mathrm{I}\sqrt{3}}{2\left(\frac{3}{2}-\frac{\mathrm{I}\sqrt{3}}{2}\right)}}$$

$$\sqrt{\frac{x}{\frac{3}{2}+\frac{\mathrm{I}\sqrt{3}}{2}}+\frac{1}{2\left(\frac{3}{2}+\frac{\mathrm{I}\sqrt{3}}{2}\right)}+\frac{\mathrm{I}\sqrt{3}}{2\left(\frac{3}{2}+\frac{\mathrm{I}\sqrt{3}}{2}\right)}}\ \mathrm{EllipticPi}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{\mathrm{I}\sqrt{3}}{2}}},\ \frac{\frac{3}{2}+\frac{\mathrm{I}\sqrt{3}}{2}}{\mathrm{I}\sqrt{2}+1},\ \sqrt{\frac{\frac{3}{2}+\frac{\mathrm{I}\sqrt{3}}{2}}{\frac{3}{2}-\frac{\mathrm{I}\sqrt{3}}{2}}}\right)\Bigg)$$

$$-\frac{1}{\sqrt{x^3-1}\,\left(\mathrm{I}\sqrt{2}+1\right)}\Bigg(\mathrm{I}\,\sqrt{\frac{x}{-\frac{3}{2}-\frac{\mathrm{I}\sqrt{3}}{2}}-\frac{1}{-\frac{3}{2}-\frac{\mathrm{I}\sqrt{3}}{2}}}\ \sqrt{\frac{x}{\frac{3}{2}-\frac{\mathrm{I}\sqrt{3}}{2}}+\frac{1}{2\left(\frac{3}{2}-\frac{\mathrm{I}\sqrt{3}}{2}\right)}-\frac{\mathrm{I}\sqrt{3}}{2\left(\frac{3}{2}-\frac{\mathrm{I}\sqrt{3}}{2}\right)}}$$

$$\sqrt{\frac{x}{\frac{3}{2}+\frac{I\sqrt{3}}{2}}+\frac{1}{2\left(\frac{3}{2}+\frac{I\sqrt{3}}{2}\right)}+\frac{I\sqrt{3}}{2\left(\frac{3}{2}+\frac{I\sqrt{3}}{2}\right)}}\ \mathrm{EllipticPi}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{I\sqrt{3}}{2}}}, \frac{\frac{3}{2}+\frac{I\sqrt{3}}{2}}{I\sqrt{2}+1}, \sqrt{\frac{\frac{3}{2}+\frac{I\sqrt{3}}{2}}{\frac{3}{2}-\frac{I\sqrt{3}}{2}}}\right)\sqrt{3}\Bigg)$$

$$-\frac{1}{\sqrt{x^3-1}\,\left(I\sqrt{2}+1\right)}\Bigg(3\,I\sqrt{2}\,\sqrt{\frac{x}{-\frac{3}{2}-\frac{I\sqrt{3}}{2}}-\frac{1}{-\frac{3}{2}-\frac{I\sqrt{3}}{2}}}\ \sqrt{\frac{x}{\frac{3}{2}-\frac{I\sqrt{3}}{2}}+\frac{1}{2\left(\frac{3}{2}-\frac{I\sqrt{3}}{2}\right)}-\frac{I\sqrt{3}}{2\left(\frac{3}{2}-\frac{I\sqrt{3}}{2}\right)}}$$

$$\sqrt{\frac{x}{\frac{3}{2}+\frac{I\sqrt{3}}{2}}+\frac{1}{2\left(\frac{3}{2}+\frac{I\sqrt{3}}{2}\right)}+\frac{I\sqrt{3}}{2\left(\frac{3}{2}+\frac{I\sqrt{3}}{2}\right)}}\ \mathrm{EllipticPi}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{I\sqrt{3}}{2}}}, \frac{\frac{3}{2}+\frac{I\sqrt{3}}{2}}{I\sqrt{2}+1}, \sqrt{\frac{\frac{3}{2}+\frac{I\sqrt{3}}{2}}{\frac{3}{2}-\frac{I\sqrt{3}}{2}}}\right)\Bigg)$$

$$+\frac{1}{\sqrt{x^3-1}\,\left(I\sqrt{2}+1\right)}\Bigg(\sqrt{2}\,\sqrt{\frac{x}{-\frac{3}{2}-\frac{I\sqrt{3}}{2}}-\frac{1}{-\frac{3}{2}-\frac{I\sqrt{3}}{2}}}\ \sqrt{\frac{x}{\frac{3}{2}-\frac{I\sqrt{3}}{2}}+\frac{1}{2\left(\frac{3}{2}-\frac{I\sqrt{3}}{2}\right)}-\frac{I\sqrt{3}}{2\left(\frac{3}{2}-\frac{I\sqrt{3}}{2}\right)}}$$

$$\sqrt{\frac{x}{\frac{3}{2}+\frac{I\sqrt{3}}{2}}+\frac{1}{2\left(\frac{3}{2}+\frac{I\sqrt{3}}{2}\right)}+\frac{I\sqrt{3}}{2\left(\frac{3}{2}+\frac{I\sqrt{3}}{2}\right)}}\ \mathrm{EllipticPi}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{I\sqrt{3}}{2}}}, \frac{\frac{3}{2}+\frac{I\sqrt{3}}{2}}{I\sqrt{2}+1}, \sqrt{\frac{\frac{3}{2}+\frac{I\sqrt{3}}{2}}{\frac{3}{2}-\frac{I\sqrt{3}}{2}}}\right)\sqrt{3}\Bigg)$$

Problem 53: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{-x^2-2x+2}{\left(dx+x^2+d+2\right)\sqrt{-x^3-1}}\,dx$$

Optimal(type 3, 26 leaves, 2 steps):

$$\frac{2\,\mathrm{arctanh}\left(\frac{(1+x)\sqrt{1+d}}{\sqrt{-x^3-1}}\right)}{\sqrt{1+d}}$$

Result(type 4, 1887 leaves):

$$\frac{2\,\mathrm{I}\sqrt{3}\,\sqrt{\mathrm{I}\left(x-\frac{1}{2}-\frac{\mathrm{I}\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{1+x}{\frac{3}{2}+\frac{\mathrm{I}\sqrt{3}}{2}}}\,\sqrt{-\mathrm{I}\left(x-\frac{1}{2}+\frac{\mathrm{I}\sqrt{3}}{2}\right)\sqrt{3}}\;\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{\mathrm{I}\left(x-\frac{1}{2}-\frac{\mathrm{I}\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{\mathrm{I}\sqrt{3}}{\frac{3}{2}+\frac{\mathrm{I}\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}$$

$$+\frac{1}{3\sqrt{d^2-4\,d-8}\,\sqrt{-x^3-1}\left(\frac{1}{2}+\frac{\mathrm{I}\sqrt{3}}{2}+\frac{d}{2}-\frac{\sqrt{d^2-4\,d-8}}{2}\right)}\left(\mathrm{I}\sqrt{3}\,\sqrt{\mathrm{I}\sqrt{3}\,x-\frac{\mathrm{I}\sqrt{3}}{2}+\frac{3}{2}}\,\sqrt{\frac{x}{\frac{3}{2}+\frac{\mathrm{I}\sqrt{3}}{2}}+\frac{1}{\frac{3}{2}+\frac{\mathrm{I}\sqrt{3}}{2}}}\,\sqrt{-\mathrm{I}\sqrt{3}\,x+\frac{\mathrm{I}\sqrt{3}}{2}+\frac{3}{2}}\;\mathrm{EllipticPi}\left(\frac{\sqrt{3}\,\sqrt{\mathrm{I}\left(x-\frac{1}{2}-\frac{\mathrm{I}\sqrt{3}}{2}\right)\sqrt{3}}}{3},\frac{\mathrm{I}\sqrt{3}}{\frac{1}{2}+\frac{\mathrm{I}\sqrt{3}}{2}+\frac{d}{2}-\frac{\sqrt{d^2-4\,d-8}}{2}},\sqrt{\frac{\mathrm{I}\sqrt{3}}{\frac{3}{2}+\frac{\mathrm{I}\sqrt{3}}{2}}}\right)d^2\right)$$

$$-\frac{1}{3\sqrt{-x^3-1}\left(\frac{1}{2}+\frac{\mathrm{I}\sqrt{3}}{2}+\frac{d}{2}-\frac{\sqrt{d^2-4\,d-8}}{2}\right)}\left(\mathrm{I}\sqrt{3}\,\sqrt{\mathrm{I}\sqrt{3}\,x-\frac{\mathrm{I}\sqrt{3}}{2}+\frac{3}{2}}\,\sqrt{\frac{x}{\frac{3}{2}+\frac{\mathrm{I}\sqrt{3}}{2}}+\frac{1}{\frac{3}{2}+\frac{\mathrm{I}\sqrt{3}}{2}}}\,\sqrt{-\mathrm{I}\sqrt{3}\,x+\frac{\mathrm{I}\sqrt{3}}{2}+\frac{3}{2}}\;\mathrm{EllipticPi}\left(\frac{\sqrt{3}\,\sqrt{\mathrm{I}\left(x-\frac{1}{2}-\frac{\mathrm{I}\sqrt{3}}{2}\right)\sqrt{3}}}{3},\frac{\mathrm{I}\sqrt{3}}{\frac{1}{2}+\frac{\mathrm{I}\sqrt{3}}{2}+\frac{d}{2}-\frac{\sqrt{d^2-4\,d-8}}{2}},\sqrt{\frac{\mathrm{I}\sqrt{3}}{\frac{3}{2}+\frac{\mathrm{I}\sqrt{3}}{2}}}\right)d\right)$$

$$-\frac{1}{3\sqrt{d^2-4\,d-8}\,\sqrt{-x^3-1}\left(\frac{1}{2}+\frac{\mathrm{I}\sqrt{3}}{2}+\frac{d}{2}-\frac{\sqrt{d^2-4\,d-8}}{2}\right)}\left(4\,\mathrm{I}\sqrt{3}\,\sqrt{\mathrm{I}\sqrt{3}\,x-\frac{\mathrm{I}\sqrt{3}}{2}+\frac{3}{2}}\,\sqrt{\frac{x}{\frac{3}{2}+\frac{\mathrm{I}\sqrt{3}}{2}}+\frac{1}{\frac{3}{2}+\frac{\mathrm{I}\sqrt{3}}{2}}}\,\sqrt{-\mathrm{I}\sqrt{3}\,x+\frac{\mathrm{I}\sqrt{3}}{2}+\frac{3}{2}}\;\mathrm{EllipticPi}\left(\frac{\sqrt{3}\,\sqrt{\mathrm{I}\left(x-\frac{1}{2}-\frac{\mathrm{I}\sqrt{3}}{2}\right)\sqrt{3}}}{3},\frac{\mathrm{I}\sqrt{3}}{\frac{1}{2}+\frac{\mathrm{I}\sqrt{3}}{2}+\frac{d}{2}-\frac{\sqrt{d^2-4\,d-8}}{2}},\sqrt{\frac{\mathrm{I}\sqrt{3}}{\frac{3}{2}+\frac{\mathrm{I}\sqrt{3}}{2}}}\right)d\right)$$

$$+\frac{1}{3\sqrt{-x^3-1}\left(\frac{1}{2}+\frac{\mathrm{I}\sqrt{3}}{2}+\frac{d}{2}-\frac{\sqrt{d^2-4\,d-8}}{2}\right)}\left(2\,\mathrm{I}\sqrt{3}\,\sqrt{\mathrm{I}\sqrt{3}\,x-\frac{\mathrm{I}\sqrt{3}}{2}+\frac{3}{2}}\,\sqrt{\frac{x}{\frac{3}{2}+\frac{\mathrm{I}\sqrt{3}}{2}}+\frac{1}{\frac{3}{2}+\frac{\mathrm{I}\sqrt{3}}{2}}}\,\sqrt{-\mathrm{I}\sqrt{3}\,x+\frac{\mathrm{I}\sqrt{3}}{2}+\frac{3}{2}}\right.$$

$$\mathrm{EllipticPi}\left(\frac{\sqrt{3}\,\sqrt{\mathrm{I}\left(x-\frac{1}{2}-\frac{\mathrm{I}\sqrt{3}}{2}\right)\sqrt{3}}}{3}, \frac{\mathrm{I}\sqrt{3}}{\frac{1}{2}+\frac{\mathrm{I}\sqrt{3}}{2}+\frac{d}{2}-\frac{\sqrt{d^2-4\,d-8}}{2}}, \sqrt{\frac{\mathrm{I}\sqrt{3}}{\frac{3}{2}+\frac{\mathrm{I}\sqrt{3}}{2}}}\right)\Bigg)$$

$$-\frac{1}{3\sqrt{d^2-4\,d-8}\,\sqrt{-x^3-1}\left(\frac{1}{2}+\frac{\mathrm{I}\sqrt{3}}{2}+\frac{d}{2}-\frac{\sqrt{d^2-4\,d-8}}{2}\right)}\Bigg(8\,\mathrm{I}\sqrt{3}\,\sqrt{\mathrm{I}\sqrt{3}\,x-\frac{\mathrm{I}\sqrt{3}}{2}+\frac{3}{2}}\,\sqrt{\frac{x}{\frac{3}{2}+\frac{\mathrm{I}\sqrt{3}}{2}}+\frac{1}{\frac{3}{2}+\frac{\mathrm{I}\sqrt{3}}{2}}}$$

$$\sqrt{-\mathrm{I}\sqrt{3}\,x+\frac{\mathrm{I}\sqrt{3}}{2}+\frac{3}{2}}\ \mathrm{EllipticPi}\left(\frac{\sqrt{3}\,\sqrt{\mathrm{I}\left(x-\frac{1}{2}-\frac{\mathrm{I}\sqrt{3}}{2}\right)\sqrt{3}}}{3}, \frac{\mathrm{I}\sqrt{3}}{\frac{1}{2}+\frac{\mathrm{I}\sqrt{3}}{2}+\frac{d}{2}-\frac{\sqrt{d^2-4\,d-8}}{2}}, \sqrt{\frac{\mathrm{I}\sqrt{3}}{\frac{3}{2}+\frac{\mathrm{I}\sqrt{3}}{2}}}\right)\Bigg)$$

$$-\frac{1}{3\sqrt{d^2-4\,d-8}\,\sqrt{-x^3-1}\left(\frac{1}{2}+\frac{\mathrm{I}\sqrt{3}}{2}+\frac{d}{2}+\frac{\sqrt{d^2-4\,d-8}}{2}\right)}\Bigg(\mathrm{I}\sqrt{3}\,\sqrt{\mathrm{I}\sqrt{3}\,x-\frac{\mathrm{I}\sqrt{3}}{2}+\frac{3}{2}}\,\sqrt{\frac{x}{\frac{3}{2}+\frac{\mathrm{I}\sqrt{3}}{2}}+\frac{1}{\frac{3}{2}+\frac{\mathrm{I}\sqrt{3}}{2}}}$$

$$\sqrt{-\mathrm{I}\sqrt{3}\,x+\frac{\mathrm{I}\sqrt{3}}{2}+\frac{3}{2}}\ \mathrm{EllipticPi}\left(\frac{\sqrt{3}\,\sqrt{\mathrm{I}\left(x-\frac{1}{2}-\frac{\mathrm{I}\sqrt{3}}{2}\right)\sqrt{3}}}{3}, \frac{\mathrm{I}\sqrt{3}}{\frac{1}{2}+\frac{\mathrm{I}\sqrt{3}}{2}+\frac{d}{2}+\frac{\sqrt{d^2-4\,d-8}}{2}}, \sqrt{\frac{\mathrm{I}\sqrt{3}}{\frac{3}{2}+\frac{\mathrm{I}\sqrt{3}}{2}}}\right)d^2\Bigg)$$

$$-\frac{1}{3\sqrt{-x^3-1}\left(\frac{1}{2}+\frac{\mathrm{I}\sqrt{3}}{2}+\frac{d}{2}+\frac{\sqrt{d^2-4\,d-8}}{2}\right)}\Bigg(\mathrm{I}\sqrt{3}\,\sqrt{\mathrm{I}\sqrt{3}\,x-\frac{\mathrm{I}\sqrt{3}}{2}+\frac{3}{2}}\,\sqrt{\frac{x}{\frac{3}{2}+\frac{\mathrm{I}\sqrt{3}}{2}}+\frac{1}{\frac{3}{2}+\frac{\mathrm{I}\sqrt{3}}{2}}}\,\sqrt{-\mathrm{I}\sqrt{3}\,x+\frac{\mathrm{I}\sqrt{3}}{2}+\frac{3}{2}}$$

$$\mathrm{EllipticPi}\left(\frac{\sqrt{3}\,\sqrt{\mathrm{I}\left(x-\frac{1}{2}-\frac{\mathrm{I}\sqrt{3}}{2}\right)\sqrt{3}}}{3}, \frac{\mathrm{I}\sqrt{3}}{\frac{1}{2}+\frac{\mathrm{I}\sqrt{3}}{2}+\frac{d}{2}+\frac{\sqrt{d^2-4\,d-8}}{2}}, \sqrt{\frac{\mathrm{I}\sqrt{3}}{\frac{3}{2}+\frac{\mathrm{I}\sqrt{3}}{2}}}\right)d\Bigg)$$

$$+\frac{1}{3\sqrt{d^2-4\,d-8}\,\sqrt{-x^3-1}\left(\frac{1}{2}+\frac{\mathrm{I}\sqrt{3}}{2}+\frac{d}{2}+\frac{\sqrt{d^2-4\,d-8}}{2}\right)}\Bigg(4\,\mathrm{I}\sqrt{3}\,\sqrt{\mathrm{I}\sqrt{3}\,x-\frac{\mathrm{I}\sqrt{3}}{2}+\frac{3}{2}}\,\sqrt{\frac{x}{\frac{3}{2}+\frac{\mathrm{I}\sqrt{3}}{2}}+\frac{1}{\frac{3}{2}+\frac{\mathrm{I}\sqrt{3}}{2}}}$$

$$\sqrt{-I\sqrt{3}\,x+\frac{I\sqrt{3}}{2}+\frac{3}{2}}\ \mathrm{EllipticPi}\left(\frac{\sqrt{3}\sqrt{I\left(x-\frac{1}{2}-\frac{I\sqrt{3}}{2}\right)\sqrt{3}}}{3},\frac{I\sqrt{3}}{\frac{1}{2}+\frac{I\sqrt{3}}{2}+\frac{d}{2}+\frac{\sqrt{d^2-4d-8}}{2}},\sqrt{\frac{I\sqrt{3}}{\frac{3}{2}+\frac{I\sqrt{3}}{2}}}\right)d\Bigg)$$

$$+\frac{1}{3\sqrt{-x^3-1}\left(\frac{1}{2}+\frac{I\sqrt{3}}{2}+\frac{d}{2}+\frac{\sqrt{d^2-4d-8}}{2}\right)}\Bigg(2I\sqrt{3}\sqrt{I\sqrt{3}\,x-\frac{I\sqrt{3}}{2}+\frac{3}{2}}\sqrt{\frac{x}{\frac{3}{2}+\frac{I\sqrt{3}}{2}}+\frac{1}{\frac{3}{2}+\frac{I\sqrt{3}}{2}}}\sqrt{-I\sqrt{3}\,x+\frac{I\sqrt{3}}{2}+\frac{3}{2}}$$

$$\mathrm{EllipticPi}\left(\frac{\sqrt{3}\sqrt{I\left(x-\frac{1}{2}-\frac{I\sqrt{3}}{2}\right)\sqrt{3}}}{3},\frac{I\sqrt{3}}{\frac{1}{2}+\frac{I\sqrt{3}}{2}+\frac{d}{2}+\frac{\sqrt{d^2-4d-8}}{2}},\sqrt{\frac{I\sqrt{3}}{\frac{3}{2}+\frac{I\sqrt{3}}{2}}}\right)\Bigg)$$

$$+\frac{1}{3\sqrt{d^2-4d-8}\sqrt{-x^3-1}\left(\frac{1}{2}+\frac{I\sqrt{3}}{2}+\frac{d}{2}+\frac{\sqrt{d^2-4d-8}}{2}\right)}\Bigg(8I\sqrt{3}\sqrt{I\sqrt{3}\,x-\frac{I\sqrt{3}}{2}+\frac{3}{2}}\sqrt{\frac{x}{\frac{3}{2}+\frac{I\sqrt{3}}{2}}+\frac{1}{\frac{3}{2}+\frac{I\sqrt{3}}{2}}}$$

$$\sqrt{-I\sqrt{3}\,x+\frac{I\sqrt{3}}{2}+\frac{3}{2}}\ \mathrm{EllipticPi}\left(\frac{\sqrt{3}\sqrt{I\left(x-\frac{1}{2}-\frac{I\sqrt{3}}{2}\right)\sqrt{3}}}{3},\frac{I\sqrt{3}}{\frac{1}{2}+\frac{I\sqrt{3}}{2}+\frac{d}{2}+\frac{\sqrt{d^2-4d-8}}{2}},\sqrt{\frac{I\sqrt{3}}{\frac{3}{2}+\frac{I\sqrt{3}}{2}}}\right)\Bigg)$$

Problem 59: Unable to integrate problem.

$$\int\frac{x^3\,(dx+c)^{1+n}}{b\,x^4+a}\,dx$$

Optimal(type 5, 293 leaves, 10 steps):

$$-\frac{(dx+c)^{n+2}\,\mathrm{hypergeom}\left([1,n+2],[3+n],\frac{b^{1/4}(dx+c)}{b^{1/4}c-(-a)^{1/4}d}\right)}{4b^{3/4}\left(b^{1/4}c-(-a)^{1/4}d\right)(n+2)}-\frac{(dx+c)^{n+2}\,\mathrm{hypergeom}\left([1,n+2],[3+n],\frac{b^{1/4}(dx+c)}{b^{1/4}c+(-a)^{1/4}d}\right)}{4b^{3/4}\left(b^{1/4}c+(-a)^{1/4}d\right)(n+2)}$$

$$-\frac{(dx+c)^{n+2}\,\mathrm{hypergeom}\left([1,n+2],[3+n],\frac{b^{1/4}(dx+c)}{b^{1/4}c-d\sqrt{-\sqrt{-a}}}\right)}{4b^{3/4}(n+2)\left(b^{1/4}c-d\sqrt{-\sqrt{-a}}\right)}-\frac{(dx+c)^{n+2}\,\mathrm{hypergeom}\left([1,n+2],[3+n],\frac{b^{1/4}(dx+c)}{b^{1/4}c+d\sqrt{-\sqrt{-a}}}\right)}{4b^{3/4}(n+2)\left(b^{1/4}c+d\sqrt{-\sqrt{-a}}\right)}$$

Result(type 8, 24 leaves):

$$\int \frac{x^3 (dx+c)^{1+n}}{bx^4+a}\, dx$$

Problem 70: Unable to integrate problem.

$$\int \frac{\left(c\sqrt{bx^2+a}\right)^{3/2}}{x^4}\, dx$$

Optimal(type 4, 151 leaves, 5 steps):

$$-\frac{\left(c\sqrt{bx^2+a}\right)^{3/2}}{3x^3} - \frac{b\left(c\sqrt{bx^2+a}\right)^{3/2}}{2ax} + \frac{b^2 x\left(c\sqrt{bx^2+a}\right)^{3/2}}{2a(bx^2+a)}$$

$$-\frac{b^{3/2}\sqrt{\cos\left(\frac{\arctan\left(\frac{x\sqrt{b}}{\sqrt{a}}\right)}{2}\right)^2}\ \mathrm{EllipticE}\left(\sin\left(\frac{\arctan\left(\frac{x\sqrt{b}}{\sqrt{a}}\right)}{2}\right), \sqrt{2}\right)\left(c\sqrt{bx^2+a}\right)^{3/2}}{2\cos\left(\frac{\arctan\left(\frac{x\sqrt{b}}{\sqrt{a}}\right)}{2}\right) a^{3/2}\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

Result(type 8, 19 leaves):

$$\int \frac{\left(c\sqrt{bx^2+a}\right)^{3/2}}{x^4}\, dx$$

Problem 72: Result more than twice size of optimal antiderivative.

$$\int x^3 \sqrt{\frac{e(bx^2+a)}{dx^2+c}}\, dx$$

Optimal(type 3, 137 leaves, 5 steps):

$$\frac{(-ad+bc)(ad+3bc)\,\mathrm{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{\sqrt{b}\sqrt{e}}\right)\sqrt{e}}{8b^{3/2}d^{5/2}} - \frac{(-ad+5bc)(dx^2+c)\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{8bd^2} + \frac{(dx^2+c)^2\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{4d^2}$$

Result(type 3, 340 leaves):

$$-\frac{1}{16\sqrt{(dx^2+c)(bx^2+a)}\, d^2 b\sqrt{bd}}\left(\sqrt{\frac{e(bx^2+a)}{dx^2+c}}\,(dx^2+c)\left(-4\sqrt{bdx^4+adx^2+bcx^2+ac}\, x^2 d b\sqrt{bd}\right.\right.$$

$$+\ln\left(\frac{2bdx^2+2\sqrt{bdx^4+adx^2+bcx^2+ac}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)a^2d^2+2\ln\left(\frac{2bdx^2+2\sqrt{bdx^4+adx^2+bcx^2+ac}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)acdb$$
$$-3b^2\ln\left(\frac{2bdx^2+2\sqrt{bdx^4+adx^2+bcx^2+ac}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)c^2-2\sqrt{bdx^4+adx^2+bcx^2+ac}\,ad\sqrt{bd}$$
$$+6\sqrt{bdx^4+adx^2+bcx^2+ac}\,cb\sqrt{bd}\Bigg)\Bigg)$$

Problem 73: Result more than twice size of optimal antiderivative.

$$\int\frac{\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{x^3}\,dx$$

Optimal(type 3, 107 leaves, 4 steps):

$$-\frac{(-ad+bc)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{\sqrt{a}\sqrt{e}}\right)\sqrt{e}}{2c^{3/2}\sqrt{a}}+\frac{(-ad+bc)\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{2c\left(a-\frac{c(bx^2+a)}{dx^2+c}\right)}$$

Result(type 3, 325 leaves):

$$\frac{1}{4\sqrt{(dx^2+c)(bx^2+a)}\,c^2ax^2\sqrt{ac}}\left(\sqrt{\frac{e(bx^2+a)}{dx^2+c}}\,(dx^2+c)\left(2bd\sqrt{bdx^4+adx^2+bcx^2+ac}\,x^4\sqrt{ac}\right.\right.$$
$$+a^2\ln\left(\frac{adx^2+bcx^2+2\sqrt{ac}\,\sqrt{bdx^4+adx^2+bcx^2+ac}+2ac}{x^2}\right)dcx^2$$
$$-\ln\left(\frac{adx^2+bcx^2+2\sqrt{ac}\,\sqrt{bdx^4+adx^2+bcx^2+ac}+2ac}{x^2}\right)bc^2ax^2+2\sqrt{bdx^4+adx^2+bcx^2+ac}\,dax^2\sqrt{ac}$$
$$\left.\left.+2\sqrt{bdx^4+adx^2+bcx^2+ac}\,bcx^2\sqrt{ac}-2(bdx^4+adx^2+bcx^2+ac)^{3/2}\sqrt{ac}\right)\right)$$

Problem 74: Result more than twice size of optimal antiderivative.

$$\int x\left(\frac{e(bx^2+a)}{dx^2+c}\right)^{3/2}dx$$

Optimal(type 3, 117 leaves, 5 steps):

$$\frac{\left(\frac{e\,(b x^2+a)}{d x^2+c}\right)^{3/2}(d x^2+c)}{2 d}-\frac{3\,(-a d+b c)\,e^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{d}\,\sqrt{\frac{e\,(b x^2+a)}{d x^2+c}}}{\sqrt{b}\,\sqrt{e}}\right)\sqrt{b}}{2 d^{5/2}}+\frac{3\,(-a d+b c)\,e\,\sqrt{\frac{e\,(b x^2+a)}{d x^2+c}}}{2 d^2}$$

Result(type 3, 431 leaves):

$$\frac{1}{4 d^2\sqrt{b d}\,\sqrt{(d x^2+c)\,(b x^2+a)}\,(b x^2+a)}\Bigg(\Bigg(3\ln\left(\frac{2 b d x^2+2\sqrt{b d x^4+a d x^2+b c x^2+a c}\,\sqrt{b d}+a d+b c}{2\sqrt{b d}}\right)x^2 a b d^2$$
$$-3\ln\left(\frac{2 b d x^2+2\sqrt{b d x^4+a d x^2+b c x^2+a c}\,\sqrt{b d}+a d+b c}{2\sqrt{b d}}\right)x^2 b^2 c d+2\sqrt{b d x^4+a d x^2+b c x^2+a c}\,x^2 d b\sqrt{b d}$$
$$+3\ln\left(\frac{2 b d x^2+2\sqrt{b d x^4+a d x^2+b c x^2+a c}\,\sqrt{b d}+a d+b c}{2\sqrt{b d}}\right)a c d b-3 b^2\ln\left(\frac{2 b d x^2+2\sqrt{b d x^4+a d x^2+b c x^2+a c}\,\sqrt{b d}+a d+b c}{2\sqrt{b d}}\right)c^2$$
$$+2\sqrt{b d x^4+a d x^2+b c x^2+a c}\,c b\sqrt{b d}-4 d\sqrt{(d x^2+c)\,(b x^2+a)}\,a\sqrt{b d}+4\sqrt{(d x^2+c)\,(b x^2+a)}\,b c\sqrt{b d}\Bigg)(d x^2+c)\left(\frac{e\,(b x^2+a)}{d x^2+c}\right)^{3/2}\Bigg)$$

Problem 75: Result more than twice size of optimal antiderivative.

$$\int\frac{\left(\frac{e\,(b x^2+a)}{d x^2+c}\right)^{3/2}}{x^5}\,dx$$

Optimal(type 3, 230 leaves, 6 steps):

$$-\frac{3\,(-5 a d+b c)\,(-a d+b c)\,e^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c}\,\sqrt{\frac{e\,(b x^2+a)}{d x^2+c}}}{\sqrt{a}\,\sqrt{e}}\right)}{8 c^{7/2}\sqrt{a}}-\frac{d\,(-a d+b c)\,e\,\sqrt{\frac{e\,(b x^2+a)}{d x^2+c}}}{c^3}-\frac{a\,(-a d+b c)^2 e^3\sqrt{\frac{e\,(b x^2+a)}{d x^2+c}}}{4 c^3\left(a e-\frac{c e\,(b x^2+a)}{d x^2+c}\right)^2}$$
$$+\frac{(-9 a d+5 b c)\,(-a d+b c)\,e^2\sqrt{\frac{e\,(b x^2+a)}{d x^2+c}}}{8 c^3\left(a e-\frac{c e\,(b x^2+a)}{d x^2+c}\right)}$$

Result(type 3, 1041 leaves):

$$-\frac{1}{16\sqrt{a c}\,a x^4 c^4\sqrt{(d x^2+c)\,(b x^2+a)}\,(b x^2+a)}\Bigg(\Bigg(18\sqrt{a c}\,\sqrt{b d x^4+a d x^2+b c x^2+a c}\,x^8 a b d^3-6\sqrt{a c}\,\sqrt{b d x^4+a d x^2+b c x^2+a c}\,x^8 b^2 c d^2$$

$$+ 15 \ln\left(\frac{a d x^2 + b c x^2 + 2 \sqrt{a c}\, \sqrt{b d x^4 + a d x^2 + b c x^2 + a c} + 2 a c}{x^2}\right) x^6 a^3 c d^3$$

$$- 18 \ln\left(\frac{a d x^2 + b c x^2 + 2 \sqrt{a c}\, \sqrt{b d x^4 + a d x^2 + b c x^2 + a c} + 2 a c}{x^2}\right) x^6 a^2 b c^2 d^2$$

$$+ 3 \ln\left(\frac{a d x^2 + b c x^2 + 2 \sqrt{a c}\, \sqrt{b d x^4 + a d x^2 + b c x^2 + a c} + 2 a c}{x^2}\right) x^6 a b^2 c^3 d + 18 \sqrt{a c}\, \sqrt{b d x^4 + a d x^2 + b c x^2 + a c}\, x^6 a^2 d^3$$

$$+ 26 \sqrt{a c}\, \sqrt{b d x^4 + a d x^2 + b c x^2 + a c}\, x^6 a b c d^2 - 12 \sqrt{a c}\, \sqrt{b d x^4 + a d x^2 + b c x^2 + a c}\, x^6 b^2 c^2 d$$

$$+ 15 \ln\left(\frac{a d x^2 + b c x^2 + 2 \sqrt{a c}\, \sqrt{b d x^4 + a d x^2 + b c x^2 + a c} + 2 a c}{x^2}\right) x^4 a^3 c^2 d^2$$

$$- 18 \ln\left(\frac{a d x^2 + b c x^2 + 2 \sqrt{a c}\, \sqrt{b d x^4 + a d x^2 + b c x^2 + a c} + 2 a c}{x^2}\right) x^4 a^2 b c^3 d$$

$$+ 3 \ln\left(\frac{a d x^2 + b c x^2 + 2 \sqrt{a c}\, \sqrt{b d x^4 + a d x^2 + b c x^2 + a c} + 2 a c}{x^2}\right) x^4 a b^2 c^4 - 18 \sqrt{a c}\, (b d x^4 + a d x^2 + b c x^2 + a c)^{3/2} x^4 a d^2 + 6 \sqrt{a c}\, (b d x^4$$

$$+ a d x^2 + b c x^2 + a c)^{3/2} x^4 b c d + 18 \sqrt{a c}\, \sqrt{b d x^4 + a d x^2 + b c x^2 + a c}\, x^4 a^2 c d^2 + 8 \sqrt{a c}\, \sqrt{b d x^4 + a d x^2 + b c x^2 + a c}\, x^4 a b c^2 d$$

$$- 6 \sqrt{a c}\, \sqrt{b d x^4 + a d x^2 + b c x^2 + a c}\, x^4 b^2 c^3 - 16 d^2 \sqrt{(d x^2 + c)\, (b x^2 + a)}\, a^2 c x^4 \sqrt{a c} + 16 d \sqrt{(d x^2 + c)\, (b x^2 + a)}\, b c^2 x^4 a \sqrt{a c}$$

$$- 14 \sqrt{a c}\, (b d x^4 + a d x^2 + b c x^2 + a c)^{3/2} x^2 a c d + 6 \sqrt{a c}\, (b d x^4 + a d x^2 + b c x^2 + a c)^{3/2} x^2 b c^2 + 4 \sqrt{a c}\, (b d x^4 + a d x^2 + b c x^2 + a c)^{3/2} a c^2 \Bigg)$$

$$(d x^2 + c) \left(\frac{e (b x^2 + a)}{d x^2 + c}\right)^{3/2} \Bigg)$$

Problem 76: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(\frac{e (b x^2 + a)}{d x^2 + c}\right)^{3/2}}{x^7}\, dx$$

Optimal(type 3, 336 leaves, 7 steps):

$$\frac{(-ad+bc)^3 e^2 \left(\frac{e(bx^2+a)}{dx^2+c}\right)^{5/2}}{6ac^2\left(ae-\frac{ce(bx^2+a)}{dx^2+c}\right)^3}+\frac{(-ad+bc)(-35a^2d^2+10bdac+c^2b^2)e^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{\sqrt{a}\sqrt{e}}\right)}{16a^{3/2}c^{9/2}}$$

$$+\frac{d^2(-ad+bc)e\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{c^4}+\frac{(-ad+bc)^2(11ad+bc)e^3\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{24c^4\left(ae-\frac{ce(bx^2+a)}{dx^2+c}\right)^2}$$

$$-\frac{(-ad+bc)(-79a^2d^2+50bdac+5c^2b^2)e^2\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{48ac^4\left(ae-\frac{ce(bx^2+a)}{dx^2+c}\right)}$$

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Result(type 3, 1497 leaves):
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$$\frac{1}{96\sqrt{ac}\,x^6a^2c^5\sqrt{(dx^2+c)(bx^2+a)}\,(bx^2+a)}\Bigg(\Bigg(105\ln\left(\frac{adx^2+bcx^2+2\sqrt{ac}\sqrt{bdx^4+adx^2+bcx^2+ac}+2ac}{x^2}\right)x^6a^4c^2d^3$$

$$+3\ln\left(\frac{adx^2+bcx^2+2\sqrt{ac}\sqrt{bdx^4+adx^2+bcx^2+ac}+2ac}{x^2}\right)x^6ab^3c^5-174\sqrt{ac}\,(bdx^4+adx^2+bcx^2+ac)^{3/2}x^6a^2d^3$$

$$-6\sqrt{ac}\sqrt{bdx^4+adx^2+bcx^2+ac}\,x^6b^3c^4+6\sqrt{ac}\,(bdx^4+adx^2+bcx^2+ac)^{3/2}x^4b^2c^3$$

$$+105\ln\left(\frac{adx^2+bcx^2+2\sqrt{ac}\sqrt{bdx^4+adx^2+bcx^2+ac}+2ac}{x^2}\right)x^8a^4cd^4+174\sqrt{ac}\sqrt{bdx^4+adx^2+bcx^2+ac}\,x^8a^3d^4$$

$$-16\sqrt{ac}\,(bdx^4+adx^2+bcx^2+ac)^{3/2}a^2c^3+3\ln\left(\frac{adx^2+bcx^2+2\sqrt{ac}\sqrt{bdx^4+adx^2+bcx^2+ac}+2ac}{x^2}\right)x^8ab^3c^4d$$

$$-12\sqrt{ac}\sqrt{bdx^4+adx^2+bcx^2+ac}\,x^8b^3c^3d-135\ln\left(\frac{adx^2+bcx^2+2\sqrt{ac}\sqrt{bdx^4+adx^2+bcx^2+ac}+2ac}{x^2}\right)x^6a^3bc^3d^2$$

$$+27\ln\left(\frac{adx^2+bcx^2+2\sqrt{ac}\sqrt{bdx^4+adx^2+bcx^2+ac}+2ac}{x^2}\right)x^6a^2b^2c^4d+6\sqrt{ac}\,(bdx^4+adx^2+bcx^2+ac)^{3/2}x^6b^2c^2d$$

$$+174\sqrt{ac}\sqrt{bdx^4+adx^2+bcx^2+ac}\,x^6a^3cd^3-96d^3\sqrt{(dx^2+c)(bx^2+a)}\,a^3cx^6\sqrt{ac}-114\sqrt{ac}\,(bdx^4+adx^2+bcx^2+ac)^{3/2}x^4a^2cd^2$$

$$+44\sqrt{ac}\,(bdx^4+adx^2+bcx^2+ac)^{3/2}x^2a^2c^2d-12\sqrt{ac}\,(bdx^4+adx^2+bcx^2+ac)^{3/2}x^2abc^3$$

$$- 6\sqrt{a c}\,\sqrt{b d x^4 + a d x^2 + b c x^2 + a c}\,x^{10} b^3 c^2 d^2 - 135 \ln\left(\frac{a d x^2 + b c x^2 + 2\sqrt{a c}\,\sqrt{b d x^4 + a d x^2 + b c x^2 + a c} + 2 a c}{x^2}\right) x^8 a^3 b c^2 d^3$$

$$+ 27 \ln\left(\frac{a d x^2 + b c x^2 + 2\sqrt{a c}\,\sqrt{b d x^4 + a d x^2 + b c x^2 + a c} + 2 a c}{x^2}\right) x^8 a^2 b^2 c^3 d^2 + 174\sqrt{a c}\,\sqrt{b d x^4 + a d x^2 + b c x^2 + a c}\,x^{10} a^2 b d^4$$

$$+ 60\sqrt{a c}\,(b d x^4 + a d x^2 + b c x^2 + a c)^{3/2} x^4 a b c^2 d - 72\sqrt{a c}\,\sqrt{b d x^4 + a d x^2 + b c x^2 + a c}\,x^{10} a b^2 c d^3$$

$$+ 216\sqrt{a c}\,\sqrt{b d x^4 + a d x^2 + b c x^2 + a c}\,x^8 a^2 b c d^3 - 138\sqrt{a c}\,\sqrt{b d x^4 + a d x^2 + b c x^2 + a c}\,x^8 a b^2 c^2 d^2 + 72\sqrt{a c}\,(b d x^4 + a d x^2 + b c x^2$$

$$+ a c)^{3/2} x^6 a b c d^2 + 42\sqrt{a c}\,\sqrt{b d x^4 + a d x^2 + b c x^2 + a c}\,x^6 a^2 b c^2 d^2 - 66\sqrt{a c}\,\sqrt{b d x^4 + a d x^2 + b c x^2 + a c}\,x^6 a b^2 c^3 d$$

$$+ 96 d^2 \sqrt{(d x^2 + c)(b x^2 + a)}\,b c^2 a^2 x^6 \sqrt{a c}\Big)\,(d x^2 + c)\left(\frac{e(b x^2 + a)}{d x^2 + c}\right)^{3/2}\Bigg)$$

Problem 77: Result more than twice size of optimal antiderivative.

$$\int x^2 \left(\frac{e(b x^2 + a)}{d x^2 + c}\right)^{3/2} dx$$

Optimal(type 4, 350 leaves, 7 steps):

$$-\frac{(-7 a d + 8 b c)\,e x\sqrt{\frac{e(b x^2 + a)}{d x^2 + c}}}{3 d^2} - \frac{e x (b x^2 + a)\sqrt{\frac{e(b x^2 + a)}{d x^2 + c}}}{d} + \frac{4 b e x (d x^2 + c)\sqrt{\frac{e(b x^2 + a)}{d x^2 + c}}}{3 d^2}$$

$$+ \frac{(-7 a d + 8 b c)\,e\sqrt{\frac{1}{1 + \frac{d x^2}{c}}}\,\sqrt{1 + \frac{d x^2}{c}}\,\mathrm{EllipticE}\left(\frac{x\sqrt{d}}{\sqrt{c}\sqrt{1 + \frac{d x^2}{c}}}, \sqrt{1 - \frac{b c}{a d}}\right)\sqrt{c}\,\sqrt{\frac{e(b x^2 + a)}{d x^2 + c}}}{3 d^{5/2}\sqrt{\frac{c(b x^2 + a)}{a(d x^2 + c)}}}$$

$$- \frac{(-3 a d + 4 b c)\,e\sqrt{\frac{1}{1 + \frac{d x^2}{c}}}\,\sqrt{1 + \frac{d x^2}{c}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{d}}{\sqrt{c}\sqrt{1 + \frac{d x^2}{c}}}, \sqrt{1 - \frac{b c}{a d}}\right)\sqrt{c}\,\sqrt{\frac{e(b x^2 + a)}{d x^2 + c}}}{3 d^{5/2}\sqrt{\frac{c(b x^2 + a)}{a(d x^2 + c)}}}$$

Result(type 4, 733 leaves):

$$\frac{1}{3 (b x^2+a)^2 d^3 \sqrt{-\frac{b}{a}} \sqrt{b d x^4+a d x^2+b c x^2+a c}} \Bigg(\left(\frac{e (b x^2+a)}{d x^2+c}\right)^{3/2} (d x^2+c) \Bigg(\sqrt{-\frac{b}{a}} \sqrt{(d x^2+c) (b x^2+a)}\, x^5 b^2 d^2$$

$$- 3 \sqrt{-\frac{b}{a}} \sqrt{b d x^4+a d x^2+b c x^2+a c}\, x^3 a b d^2 + 3 \sqrt{-\frac{b}{a}} \sqrt{b d x^4+a d x^2+b c x^2+a c}\, x^3 b^2 c d + \sqrt{-\frac{b}{a}} \sqrt{(d x^2+c) (b x^2+a)}\, x^3 a b d^2$$

$$+ \sqrt{-\frac{b}{a}} \sqrt{(d x^2+c) (b x^2+a)}\, x^3 b^2 c d + 3 \sqrt{\frac{b x^2+a}{a}} \sqrt{\frac{d x^2+c}{c}} \operatorname{EllipticF}\left(x \sqrt{-\frac{b}{a}}, \sqrt{\frac{a d}{b c}}\right) \sqrt{(d x^2+c) (b x^2+a)}\, a^2 d^2$$

$$- 11 \sqrt{\frac{b x^2+a}{a}} \sqrt{\frac{d x^2+c}{c}} \operatorname{EllipticF}\left(x \sqrt{-\frac{b}{a}}, \sqrt{\frac{a d}{b c}}\right) \sqrt{(d x^2+c) (b x^2+a)}\, a b c d + 8 \sqrt{\frac{b x^2+a}{a}} \sqrt{\frac{d x^2+c}{c}} \operatorname{EllipticF}\left(x \sqrt{-\frac{b}{a}},\right.$$

$$\left.\sqrt{\frac{a d}{b c}}\right) \sqrt{(d x^2+c) (b x^2+a)}\, b^2 c^2 + 7 \sqrt{\frac{b x^2+a}{a}} \sqrt{\frac{d x^2+c}{c}} \operatorname{EllipticE}\left(x \sqrt{-\frac{b}{a}}, \sqrt{\frac{a d}{b c}}\right) \sqrt{(d x^2+c) (b x^2+a)}\, a b c d$$

$$- 8 \sqrt{\frac{b x^2+a}{a}} \sqrt{\frac{d x^2+c}{c}} \operatorname{EllipticE}\left(x \sqrt{-\frac{b}{a}}, \sqrt{\frac{a d}{b c}}\right) \sqrt{(d x^2+c) (b x^2+a)}\, b^2 c^2 - 3 \sqrt{-\frac{b}{a}} \sqrt{b d x^4+a d x^2+b c x^2+a c}\, x a^2 d^2$$

$$+ 3 \sqrt{-\frac{b}{a}} \sqrt{b d x^4+a d x^2+b c x^2+a c}\, x a b c d + \sqrt{-\frac{b}{a}} \sqrt{(d x^2+c) (b x^2+a)}\, x a b c d \Bigg) \Bigg)$$

Problem 80: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x \sqrt{\frac{e (b x^2+a)}{d x^2+c}}} \, dx$$

Optimal(type 3, 84 leaves, 5 steps):

$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c} \sqrt{\frac{e (b x^2+a)}{d x^2+c}}}{\sqrt{a} \sqrt{e}}\right) \sqrt{c}}{\sqrt{a} \sqrt{e}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{d} \sqrt{\frac{e (b x^2+a)}{d x^2+c}}}{\sqrt{b} \sqrt{e}}\right) \sqrt{d}}{\sqrt{b} \sqrt{e}}$$

Result(type 3, 178 leaves):

$$-\frac{1}{2 \sqrt{\frac{e (b x^2+a)}{d x^2+c}} \sqrt{(d x^2+c) (b x^2+a)} \sqrt{b d} \sqrt{a c}} \Bigg((b x^2+a) \Bigg(c \ln\left(\frac{a d x^2+b c x^2+2 \sqrt{a c} \sqrt{b d x^4+a d x^2+b c x^2+a c}+2 a c}{x^2}\right) \sqrt{b d}$$

$$- \ln\left(\frac{2 b d x^2+2 \sqrt{b d x^4+a d x^2+b c x^2+a c} \sqrt{b d}+a d+b c}{2 \sqrt{b d}}\right) d \sqrt{a c} \Bigg) \Bigg)$$

Problem 82: Result more than twice size of optimal antiderivative.

$$\int \frac{x}{\left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2}}\,dx$$

Optimal(type 3, 122 leaves, 5 steps):

$$\frac{3\,(-a\,d+b\,c)\,\operatorname{arctanh}\left(\frac{\sqrt{d}\,\sqrt{\frac{e\,(b\,x^2+a)}{d\,x^2+c}}}{\sqrt{b}\,\sqrt{e}}\right)\sqrt{d}}{2\,b^{5/2}\,e^{3/2}}-\frac{3\,(-a\,d+b\,c)}{2\,b^2\,e\,\sqrt{\frac{e\,(b\,x^2+a)}{d\,x^2+c}}}+\frac{d\,x^2+c}{2\,b\,e\,\sqrt{\frac{e\,(b\,x^2+a)}{d\,x^2+c}}}$$

Result(type 3, 431 leaves):

$$-\frac{1}{4\,b^2\,\sqrt{b\,d}\,\sqrt{(d\,x^2+c)\,(b\,x^2+a)}\,(d\,x^2+c)\left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2}}\Bigg(\Bigg(3\ln\left(\frac{2\,b\,d\,x^2+2\,\sqrt{b\,d\,x^4+a\,d\,x^2+b\,c\,x^2+a\,c}\,\sqrt{b\,d}+a\,d+b\,c}{2\,\sqrt{b\,d}}\right)x^2\,a\,b\,d^2$$
$$-3\ln\left(\frac{2\,b\,d\,x^2+2\,\sqrt{b\,d\,x^4+a\,d\,x^2+b\,c\,x^2+a\,c}\,\sqrt{b\,d}+a\,d+b\,c}{2\,\sqrt{b\,d}}\right)x^2\,b^2\,c\,d-2\,\sqrt{b\,d\,x^4+a\,d\,x^2+b\,c\,x^2+a\,c}\,x^2\,d\,b\,\sqrt{b\,d}$$
$$+3\ln\left(\frac{2\,b\,d\,x^2+2\,\sqrt{b\,d\,x^4+a\,d\,x^2+b\,c\,x^2+a\,c}\,\sqrt{b\,d}+a\,d+b\,c}{2\,\sqrt{b\,d}}\right)a^2\,d^2-3\ln\left(\frac{2\,b\,d\,x^2+2\,\sqrt{b\,d\,x^4+a\,d\,x^2+b\,c\,x^2+a\,c}\,\sqrt{b\,d}+a\,d+b\,c}{2\,\sqrt{b\,d}}\right)a\,c\,d\,b$$
$$-2\,\sqrt{b\,d\,x^4+a\,d\,x^2+b\,c\,x^2+a\,c}\,a\,d\,\sqrt{b\,d}-4\,d\,\sqrt{(d\,x^2+c)\,(b\,x^2+a)}\,a\,\sqrt{b\,d}+4\,\sqrt{(d\,x^2+c)\,(b\,x^2+a)}\,b\,c\,\sqrt{b\,d}\Bigg)(b\,x^2+a)\Bigg)$$

Problem 83: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^5\left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2}}\,dx$$

Optimal(type 3, 229 leaves, 6 steps):

$$-\frac{3\,(-a\,d+b\,c)\,(-a\,d+5\,b\,c)\,\operatorname{arctanh}\left(\frac{\sqrt{c}\,\sqrt{\frac{e\,(b\,x^2+a)}{d\,x^2+c}}}{\sqrt{a}\,\sqrt{e}}\right)}{8\,a^{7/2}\,e^{3/2}\,\sqrt{c}}+\frac{b\,(-a\,d+b\,c)}{a^3\,e\,\sqrt{\frac{e\,(b\,x^2+a)}{d\,x^2+c}}}-\frac{(-a\,d+b\,c)^2\,\sqrt{\frac{e\,(b\,x^2+a)}{d\,x^2+c}}}{4\,a^2\left(a\,e-\frac{c\,e\,(b\,x^2+a)}{d\,x^2+c}\right)^2}$$

$$-\frac{(-3ad+7bc)(-ad+bc)\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{8a^3\left(ae^2-\frac{ce^2(bx^2+a)}{dx^2+c}\right)}$$

Result(type 3, 1041 leaves):

$$-\frac{1}{16\sqrt{ac}\,cx^4a^4\sqrt{(dx^2+c)(bx^2+a)}\,(dx^2+c)\left(\frac{e(bx^2+a)}{dx^2+c}\right)^{3/2}}\Bigg(\Bigg(-6\sqrt{ac}\sqrt{bdx^4+adx^2+bcx^2+ac}\,x^8ab^2d^2$$

$$+18\sqrt{ac}\sqrt{bdx^4+adx^2+bcx^2+ac}\,x^8b^3cd+3\ln\left(\frac{adx^2+bcx^2+2\sqrt{ac}\sqrt{bdx^4+adx^2+bcx^2+ac}+2ac}{x^2}\right)x^6a^3bcd^2$$

$$-18\ln\left(\frac{adx^2+bcx^2+2\sqrt{ac}\sqrt{bdx^4+adx^2+bcx^2+ac}+2ac}{x^2}\right)x^6a^2b^2c^2d$$

$$+15\ln\left(\frac{adx^2+bcx^2+2\sqrt{ac}\sqrt{bdx^4+adx^2+bcx^2+ac}+2ac}{x^2}\right)x^6ab^3c^3-12\sqrt{ac}\sqrt{bdx^4+adx^2+bcx^2+ac}\,x^6a^2bd^2$$

$$+26\sqrt{ac}\sqrt{bdx^4+adx^2+bcx^2+ac}\,x^6ab^2cd+18\sqrt{ac}\sqrt{bdx^4+adx^2+bcx^2+ac}\,x^6b^3c^2$$

$$+3\ln\left(\frac{adx^2+bcx^2+2\sqrt{ac}\sqrt{bdx^4+adx^2+bcx^2+ac}+2ac}{x^2}\right)x^4a^4cd^2$$

$$-18\ln\left(\frac{adx^2+bcx^2+2\sqrt{ac}\sqrt{bdx^4+adx^2+bcx^2+ac}+2ac}{x^2}\right)x^4a^3bc^2d$$

$$+15\ln\left(\frac{adx^2+bcx^2+2\sqrt{ac}\sqrt{bdx^4+adx^2+bcx^2+ac}+2ac}{x^2}\right)x^4a^2b^2c^3+6\sqrt{ac}\,(bdx^4+adx^2+bcx^2+ac)^{3/2}x^4abd$$

$$-18\sqrt{ac}\,(bdx^4+adx^2+bcx^2+ac)^{3/2}x^4b^2c-6\sqrt{ac}\sqrt{bdx^4+adx^2+bcx^2+ac}\,x^4a^3d^2+8\sqrt{ac}\sqrt{bdx^4+adx^2+bcx^2+ac}\,x^4a^2bcd$$

$$+18\sqrt{ac}\sqrt{bdx^4+adx^2+bcx^2+ac}\,x^4ab^2c^2+16d\sqrt{(dx^2+c)(bx^2+a)}\,ba^2x^4c\sqrt{ac}-16b^2\sqrt{(dx^2+c)(bx^2+a)}\,c^2ax^4\sqrt{ac}$$

$$+6\sqrt{ac}\,(bdx^4+adx^2+bcx^2+ac)^{3/2}x^2a^2d-14\sqrt{ac}\,(bdx^4+adx^2+bcx^2+ac)^{3/2}x^2abc+4\sqrt{ac}\,(bdx^4+adx^2+bcx^2+ac)^{3/2}a^2c\Bigg)$$

$$(bx^2+a)\Bigg)$$

Problem 87: Result more than twice size of optimal antiderivative.

$$\int x^3 \left(a+\frac{b}{d x^2+c}\right)^{3/2} dx$$

Optimal(type 3, 154 leaves, 7 steps):

$$\frac{3 b (-4 a c+b) \operatorname{arctanh}\left(\frac{\sqrt{\frac{a d x^2+a c+b}{d x^2+c}}}{\sqrt{a}}\right)}{8 d^2 \sqrt{a}}+\frac{b c \sqrt{\frac{a d x^2+a c+b}{d x^2+c}}}{d^2}+\frac{(-4 a c+5 b) (d x^2+c) \sqrt{\frac{a d x^2+a c+b}{d x^2+c}}}{8 d^2}+\frac{a (d x^2+c)^2 \sqrt{\frac{a d x^2+a c+b}{d x^2+c}}}{4 d^2}$$

Result(type 3, 592 leaves):

$$\frac{1}{16 d^2 \sqrt{a d^2}\,\sqrt{(d x^2+c)(a d x^2+a c+b)}}\Bigg(\Bigg(4 \sqrt{a d^2}\,\sqrt{x^4 a d^2+2 x^2 a c d+b d x^2+c^2 a+b c}\, x^4 a d^2 - 12 \ln\left(\frac{2 a d^2 x^2+2 a c d+2 \sqrt{x^4 a d^2+2 x^2 a c d+b d x^2+c^2 a+b c}\,\sqrt{a d^2}+b d}{2 \sqrt{a d^2}}\right) x^2 a b c d^2 + 3 \ln\left(\frac{2 a d^2 x^2+2 a c d+2 \sqrt{x^4 a d^2+2 x^2 a c d+b d x^2+c^2 a+b c}\,\sqrt{a d^2}+b d}{2 \sqrt{a d^2}}\right) x^2 b^2 d^2 + 10 \sqrt{a d^2}\,\sqrt{x^4 a d^2+2 x^2 a c d+b d x^2+c^2 a+b c}\, x^2 b d - 12 \ln\left(\frac{2 a d^2 x^2+2 a c d+2 \sqrt{x^4 a d^2+2 x^2 a c d+b d x^2+c^2 a+b c}\,\sqrt{a d^2}+b d}{2 \sqrt{a d^2}}\right) a b c^2 d - 4 \sqrt{a d^2}\,\sqrt{x^4 a d^2+2 x^2 a c d+b d x^2+c^2 a+b c}\, a c^2 + 3 \ln\left(\frac{2 a d^2 x^2+2 a c d+2 \sqrt{x^4 a d^2+2 x^2 a c d+b d x^2+c^2 a+b c}\,\sqrt{a d^2}+b d}{2 \sqrt{a d^2}}\right) b^2 c d + 16 b c \sqrt{(d x^2+c)(a d x^2+a c+b)}\,\sqrt{a d^2} + 10 \sqrt{a d^2}\,\sqrt{x^4 a d^2+2 x^2 a c d+b d x^2+c^2 a+b c}\, b c\Bigg) \sqrt{\frac{a d x^2+a c+b}{d x^2+c}}\Bigg)$$

Problem 88: Result more than twice size of optimal antiderivative.

$$\int x \left(a+\frac{b}{d x^2+c}\right)^{3/2} dx$$

Optimal(type 3, 78 leaves, 6 steps):

$$\frac{(d x^2+c)\left(a+\frac{b}{d x^2+c}\right)^{3/2}}{2 d}+\frac{3 b \operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{d x^2+c}}}{\sqrt{a}}\right)\sqrt{a}}{2 d}-\frac{3 b \sqrt{a+\frac{b}{d x^2+c}}}{2 d}$$

Result(type 3, 335 leaves):

$$-\frac{1}{4 d \sqrt{a d^2}\,\sqrt{(d x^2+c)(a d x^2+a c+b)}}\Bigg(\Bigg(-3 \ln\left(\frac{2 a d^2 x^2+2 a c d+2\sqrt{x^4 a d^2+2 x^2 a c d+b d x^2+c^2 a+b c}\,\sqrt{a d^2}+b d}{2\sqrt{a d^2}}\right) x^2 a b d^2$$
$$-2 a \sqrt{x^4 a d^2+2 x^2 a c d+b d x^2+c^2 a+b c}\, x^2 d \sqrt{a d^2}-3 b \ln\left(\frac{2 a d^2 x^2+2 a c d+2\sqrt{x^4 a d^2+2 x^2 a c d+b d x^2+c^2 a+b c}\,\sqrt{a d^2}+b d}{2\sqrt{a d^2}}\right) a c d$$
$$-2 a \sqrt{x^4 a d^2+2 x^2 a c d+b d x^2+c^2 a+b c}\, c \sqrt{a d^2}+4 b \sqrt{(d x^2+c)(a d x^2+a c+b)}\,\sqrt{a d^2}\Bigg)\sqrt{\frac{a d x^2+a c+b}{d x^2+c}}\Bigg)$$

Problem 89: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(a+\frac{b}{d x^2+c}\right)^{3/2}}{x^3}\,dx$$

Optimal(type 3, 118 leaves, 6 steps):

$$-\frac{(d x^2+c)\left(\frac{a d x^2+a c+b}{d x^2+c}\right)^{3/2}}{2 c x^2}+\frac{3 b d \operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{a d x^2+a c+b}{d x^2+c}}}{\sqrt{a c+b}}\right)\sqrt{a c+b}}{2 c^{5/2}}-\frac{3 b d \sqrt{\frac{a d x^2+a c+b}{d x^2+c}}}{2 c^2}$$

Result(type 3, 819 leaves):

$$-\frac{1}{4\sqrt{c^2 a+b c}\, x^2 c^3 \sqrt{(d x^2+c)(a d x^2+a c+b)}}\Bigg(\sqrt{\frac{a d x^2+a c+b}{d x^2+c}}\Bigg(-2\sqrt{c^2 a+b c}\,\sqrt{x^4 a d^2+2 x^2 a c d+b d x^2+c^2 a+b c}\, x^6 a d^3$$
$$-3 \ln\left(\frac{2 x^2 a c d+b d x^2+2 c^2 a+2\sqrt{c^2 a+b c}\,\sqrt{x^4 a d^2+2 x^2 a c d+b d x^2+c^2 a+b c}+2 b c}{x^2}\right) x^4 a b c^2 d^2$$
$$-6\sqrt{c^2 a+b c}\,\sqrt{x^4 a d^2+2 x^2 a c d+b d x^2+c^2 a+b c}\, x^4 a c d^2$$
$$-3 \ln\left(\frac{2 x^2 a c d+b d x^2+2 c^2 a+2\sqrt{c^2 a+b c}\,\sqrt{x^4 a d^2+2 x^2 a c d+b d x^2+c^2 a+b c}+2 b c}{x^2}\right) x^4 b^2 c d^2$$

$$- 2 \sqrt{c^2 a + b c}\, \sqrt{x^4 a d^2 + 2 x^2 a c d + b d x^2 + c^2 a + b c}\, x^4 b d^2$$

$$- 3 \ln\left(\frac{2 x^2 a c d + b d x^2 + 2 c^2 a + 2 \sqrt{c^2 a + b c}\, \sqrt{x^4 a d^2 + 2 x^2 a c d + b d x^2 + c^2 a + b c} + 2 b c}{x^2}\right) x^2 a b c^3 d$$

$$- 4 \sqrt{c^2 a + b c}\, \sqrt{x^4 a d^2 + 2 x^2 a c d + b d x^2 + c^2 a + b c}\, x^2 a c^2 d$$

$$- 3 \ln\left(\frac{2 x^2 a c d + b d x^2 + 2 c^2 a + 2 \sqrt{c^2 a + b c}\, \sqrt{x^4 a d^2 + 2 x^2 a c d + b d x^2 + c^2 a + b c} + 2 b c}{x^2}\right) x^2 b^2 c^2 d$$

$$+ 4 \sqrt{(d x^2 + c) (a d x^2 + a c + b)}\, \sqrt{c^2 a + b c}\, x^2 b c d + 2 \sqrt{c^2 a + b c}\, (x^4 a d^2 + 2 x^2 a c d + b d x^2 + c^2 a + b c)^{3/2} x^2 d$$

$$- 2 \sqrt{c^2 a + b c}\, \sqrt{x^4 a d^2 + 2 x^2 a c d + b d x^2 + c^2 a + b c}\, x^2 b c d + 2 \sqrt{c^2 a + b c}\, (x^4 a d^2 + 2 x^2 a c d + b d x^2 + c^2 a + b c)^{3/2} c \Big) \Big)$$

Problem 90: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(a + \frac{b}{d x^2 + c}\right)^{3/2}}{x^7}\, dx$$

Optimal(type 3, 266 leaves, 8 steps):

$$-\frac{(d x^2 + c)^3 \left(\frac{a d x^2 + a c + b}{d x^2 + c}\right)^{5/2}}{6 c^2 (a c + b) x^6} + \frac{b (24 a^2 c^2 + 60 a b c + 35 b^2) d^3 \operatorname{arctanh}\left(\frac{\sqrt{c}\, \sqrt{\frac{a d x^2 + a c + b}{d x^2 + c}}}{\sqrt{a c + b}}\right)}{16 c^{9/2} (a c + b)^{3/2}} - \frac{b d^3 \sqrt{\frac{a d x^2 + a c + b}{d x^2 + c}}}{c^4}$$

$$- \frac{(24 a^2 c^2 + 108 a b c + 79 b^2) d^2 (d x^2 + c) \sqrt{\frac{a d x^2 + a c + b}{d x^2 + c}}}{48 c^4 (a c + b) x^2} + \frac{(12 a c + 11 b) d (d x^2 + c)^2 \sqrt{\frac{a d x^2 + a c + b}{d x^2 + c}}}{24 c^4 x^4}$$

Result(type ?, 2604 leaves): Display of huge result suppressed!

Problem 91: Result more than twice size of optimal antiderivative.

$$\int x^2 \left(a + \frac{b}{d x^2 + c}\right)^{3/2} dx$$

Optimal(type 4, 371 leaves, 8 steps):

$$\frac{(-a c+7 b) x \sqrt{\frac{a d x^2+a c+b}{d x^2+c}}}{3 d}+\frac{4 a x (d x^2+c) \sqrt{\frac{a d x^2+a c+b}{d x^2+c}}}{3 d}-\frac{x (a d x^2+a c+b) \sqrt{\frac{a d x^2+a c+b}{d x^2+c}}}{d}$$

$$-\frac{(-a c+7 b) \sqrt{\frac{1}{1+\frac{d x^2}{c}}} \sqrt{1+\frac{d x^2}{c}}\, \mathrm{EllipticE}\left(\frac{x \sqrt{d}}{\sqrt{c} \sqrt{1+\frac{d x^2}{c}}}, \sqrt{\frac{b}{a c+b}}\right) \sqrt{c} \sqrt{\frac{a d x^2+a c+b}{d x^2+c}}}{3 d^{3/2} \sqrt{\frac{c (a d x^2+a c+b)}{(a c+b) (d x^2+c)}}}$$

$$+\frac{(-a c+3 b) \sqrt{\frac{1}{1+\frac{d x^2}{c}}} \sqrt{1+\frac{d x^2}{c}}\, \mathrm{EllipticF}\left(\frac{x \sqrt{d}}{\sqrt{c} \sqrt{1+\frac{d x^2}{c}}}, \sqrt{\frac{b}{a c+b}}\right) \sqrt{c} \sqrt{\frac{a d x^2+a c+b}{d x^2+c}}}{3 d^{3/2} \sqrt{\frac{c (a d x^2+a c+b)}{(a c+b) (d x^2+c)}}}$$

Result(type 4, 822 leaves):

$$-\frac{1}{3 d \sqrt{x^4 a d^2+2 x^2 a c d+b d x^2+c^2 a+b c} \sqrt{-\frac{a d}{a c+b}} (a d x^2+a c+b)} \Bigg(\Bigg(-\sqrt{(d x^2+c) (a d x^2+a c+b)} \sqrt{-\frac{a d}{a c+b}}\, x^5 a^2 d^2$$

$$-2 \sqrt{(d x^2+c) (a d x^2+a c+b)} \sqrt{-\frac{a d}{a c+b}}\, x^3 a^2 c d+\sqrt{\frac{d x^2+c}{c}} \sqrt{(d x^2+c) (a d x^2+a c+b)} \sqrt{\frac{a d x^2+a c+b}{a c+b}}\, \mathrm{EllipticE}\left(x \sqrt{-\frac{a d}{a c+b}},\right.$$

$$\left.\sqrt{\frac{a c+b}{a c}}\right) a^2 c^2-\sqrt{(d x^2+c) (a d x^2+a c+b)} \sqrt{-\frac{a d}{a c+b}}\, x^3 a b d+3 \sqrt{x^4 a d^2+2 x^2 a c d+b d x^2+c^2 a+b c} \sqrt{-\frac{a d}{a c+b}}\, x^3 a b d$$

$$+5 \sqrt{\frac{d x^2+c}{c}} \sqrt{(d x^2+c) (a d x^2+a c+b)} \sqrt{\frac{a d x^2+a c+b}{a c+b}}\, \mathrm{EllipticF}\left(x \sqrt{-\frac{a d}{a c+b}}, \sqrt{\frac{a c+b}{a c}}\right) a b c$$

$$-7 \sqrt{\frac{d x^2+c}{c}} \sqrt{(d x^2+c) (a d x^2+a c+b)} \sqrt{\frac{a d x^2+a c+b}{a c+b}}\, \mathrm{EllipticE}\left(x \sqrt{-\frac{a d}{a c+b}}, \sqrt{\frac{a c+b}{a c}}\right) a b c$$

$$-\sqrt{(d x^2+c) (a d x^2+a c+b)} \sqrt{-\frac{a d}{a c+b}}\, x a^2 c^2-3 \sqrt{\frac{d x^2+c}{c}} \sqrt{(d x^2+c) (a d x^2+a c+b)} \sqrt{\frac{a d x^2+a c+b}{a c+b}}\, \mathrm{EllipticF}\left(x \sqrt{-\frac{a d}{a c+b}},\right.$$

$$\left.\sqrt{\frac{a c+b}{a c}}\right) b^2-\sqrt{(d x^2+c) (a d x^2+a c+b)} \sqrt{-\frac{a d}{a c+b}}\, x a b c+3 \sqrt{x^4 a d^2+2 x^2 a c d+b d x^2+c^2 a+b c} \sqrt{-\frac{a d}{a c+b}}\, x a b c$$

$$+3 \sqrt{x^4 a d^2+2 x^2 a c d+b d x^2+c^2 a+b c} \sqrt{-\frac{a d}{a c+b}}\, x b^2\Bigg) \sqrt{\frac{a d x^2+a c+b}{d x^2+c}}\Bigg)$$

Problem 92: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(a+\frac{b}{d x^2+c}\right)^{3/2}}{x^6}\, dx$$

Optimal(type 4, 526 leaves, 10 steps):

$$\frac{b\sqrt{\frac{a d x^2+a c+b}{d x^2+c}}}{c x^5}+\frac{(a^2c^2+16abc+16b^2)d^3x\sqrt{\frac{a d x^2+a c+b}{d x^2+c}}}{5c^4(ac+b)}-\frac{(ac+6b)(dx^2+c)\sqrt{\frac{a d x^2+a c+b}{d x^2+c}}}{5c^2x^5}$$
$$+\frac{(ac+8b)d(dx^2+c)\sqrt{\frac{a d x^2+a c+b}{d x^2+c}}}{5c^3x^3}-\frac{(a^2c^2+16abc+16b^2)d^2(dx^2+c)\sqrt{\frac{a d x^2+a c+b}{d x^2+c}}}{5c^4(ac+b)x}$$
$$-\frac{(a^2c^2+16abc+16b^2)d^{5/2}\sqrt{\frac{1}{1+\frac{dx^2}{c}}}\sqrt{1+\frac{dx^2}{c}}\,\mathrm{EllipticE}\left(\frac{x\sqrt{d}}{\sqrt{c}\sqrt{1+\frac{dx^2}{c}}},\sqrt{\frac{b}{ac+b}}\right)\sqrt{\frac{a d x^2+a c+b}{d x^2+c}}}{5c^{7/2}(ac+b)\sqrt{\frac{c(adx^2+ac+b)}{(ac+b)(dx^2+c)}}}$$
$$+\frac{a(ac+8b)d^{5/2}\sqrt{\frac{1}{1+\frac{dx^2}{c}}}\sqrt{1+\frac{dx^2}{c}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{d}}{\sqrt{c}\sqrt{1+\frac{dx^2}{c}}},\sqrt{\frac{b}{ac+b}}\right)\sqrt{\frac{a d x^2+a c+b}{d x^2+c}}}{5c^{5/2}(ac+b)\sqrt{\frac{c(adx^2+ac+b)}{(ac+b)(dx^2+c)}}}$$

Result(type 4, 1665 leaves):

$$-\Bigg(\Bigg(5\sqrt{x^4ad^2+2x^2acd+bdx^2+c^2a+bc}\sqrt{-\frac{ad}{ac+b}}\,x^6b^3d^3+11\sqrt{(dx^2+c)(adx^2+ac+b)}\sqrt{-\frac{ad}{ac+b}}\,x^6b^3d^3$$
$$+3\sqrt{(dx^2+c)(adx^2+ac+b)}\sqrt{-\frac{ad}{ac+b}}\,a^2bc^5+3\sqrt{(dx^2+c)(adx^2+ac+b)}\sqrt{-\frac{ad}{ac+b}}\,ab^2c^4$$
$$-\sqrt{\frac{dx^2+c}{c}}\sqrt{(dx^2+c)(adx^2+ac+b)}\sqrt{\frac{adx^2+ac+b}{ac+b}}\,\mathrm{EllipticE}\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{\frac{ac+b}{ac}}\right)x^5a^3c^3d^3$$
$$+11\sqrt{(dx^2+c)(adx^2+ac+b)}\sqrt{-\frac{ad}{ac+b}}\,x^8a^2bcd^4+19\sqrt{(dx^2+c)(adx^2+ac+b)}\sqrt{-\frac{ad}{ac+b}}\,x^6a^2bc^2d^3$$
$$+30\sqrt{(dx^2+c)(adx^2+ac+b)}\sqrt{-\frac{ad}{ac+b}}\,x^6ab^2cd^3+5\sqrt{(dx^2+c)(adx^2+ac+b)}\sqrt{-\frac{ad}{ac+b}}\,x^4a^2bc^3d^2$$

$$+ 13\sqrt{(dx^2+c)(adx^2+ac+b)}\sqrt{-\frac{ad}{ac+b}}\,x^4ab^2c^2d^2 - 3\sqrt{(dx^2+c)(adx^2+ac+b)}\sqrt{-\frac{ad}{ac+b}}\,x^2ab^2c^3d$$

$$+ 5\sqrt{x^4ad^2+2x^2acd+bdx^2+c^2a+bc}\sqrt{-\frac{ad}{ac+b}}\,x^8a^2bcd^4 + 5\sqrt{x^4ad^2+2x^2acd+bdx^2+c^2a+bc}\sqrt{-\frac{ad}{ac+b}}\,x^6a^2bc^2d^3$$

$$+ 10\sqrt{x^4ad^2+2x^2acd+bdx^2+c^2a+bc}\sqrt{-\frac{ad}{ac+b}}\,x^6ab^2cd^3 + \sqrt{(dx^2+c)(adx^2+ac+b)}\sqrt{-\frac{ad}{ac+b}}\,a^3c^6$$

$$+ \sqrt{(dx^2+c)(adx^2+ac+b)}\sqrt{-\frac{ad}{ac+b}}\,b^3c^3 + 7\sqrt{\frac{dx^2+c}{c}}\sqrt{(dx^2+c)(adx^2+ac+b)}\sqrt{\frac{adx^2+ac+b}{ac+b}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{ad}{ac+b}},\right.$$

$$\left.\sqrt{\frac{ac+b}{ac}}\right)x^5a^2bc^2d^3 - 16\sqrt{\frac{dx^2+c}{c}}\sqrt{(dx^2+c)(adx^2+ac+b)}\sqrt{\frac{adx^2+ac+b}{ac+b}}\,\mathrm{EllipticE}\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{\frac{ac+b}{ac}}\right)x^5a^2bc^2d^3$$

$$+ 8\sqrt{\frac{dx^2+c}{c}}\sqrt{(dx^2+c)(adx^2+ac+b)}\sqrt{\frac{adx^2+ac+b}{ac+b}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{\frac{ac+b}{ac}}\right)x^5ab^2cd^3$$

$$- 16\sqrt{\frac{dx^2+c}{c}}\sqrt{(dx^2+c)(adx^2+ac+b)}\sqrt{\frac{adx^2+ac+b}{ac+b}}\,\mathrm{EllipticE}\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{\frac{ac+b}{ac}}\right)x^5ab^2cd^3$$

$$+ \sqrt{(dx^2+c)(adx^2+ac+b)}\sqrt{-\frac{ad}{ac+b}}\,x^6a^3c^3d^3 + \sqrt{(dx^2+c)(adx^2+ac+b)}\sqrt{-\frac{ad}{ac+b}}\,x^2a^3c^5d$$

$$+ 8\sqrt{(dx^2+c)(adx^2+ac+b)}\sqrt{-\frac{ad}{ac+b}}\,x^4b^3cd^2 - 2\sqrt{(dx^2+c)(adx^2+ac+b)}\sqrt{-\frac{ad}{ac+b}}\,x^2b^3c^2d$$

$$+ 5\sqrt{x^4ad^2+2x^2acd+bdx^2+c^2a+bc}\sqrt{-\frac{ad}{ac+b}}\,x^8ab^2d^4 + \sqrt{(dx^2+c)(adx^2+ac+b)}\sqrt{-\frac{ad}{ac+b}}\,x^8a^3c^2d^4$$

$$\left.+ 11\sqrt{(dx^2+c)(adx^2+ac+b)}\sqrt{-\frac{ad}{ac+b}}\,x^8ab^2d^4\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\Bigg/\left(5\sqrt{x^4ad^2+2x^2acd+bdx^2+c^2a+bc}\sqrt{-\frac{ad}{ac+b}}\,(ac\right.$$

$$\left.+b)\,x^5c^4\,(adx^2+ac+b)\right)$$

Problem 93: Result more than twice size of optimal antiderivative.

$$\int\frac{x^3}{\sqrt{a+\frac{b}{dx^2+c}}}\,dx$$

Optimal(type 3, 132 leaves, 6 steps):

$$\frac{b\,(4\,a\,c+3\,b)\,\operatorname{arctanh}\left(\frac{\sqrt{\frac{a\,d\,x^2+a\,c+b}{d\,x^2+c}}}{\sqrt{a}}\right)}{8\,a^{5/2}\,d^2}-\frac{(4\,a\,c+3\,b)\,(d\,x^2+c)\,\sqrt{\frac{a\,d\,x^2+a\,c+b}{d\,x^2+c}}}{8\,a^2\,d^2}+\frac{(d\,x^2+c)^2\,\sqrt{\frac{a\,d\,x^2+a\,c+b}{d\,x^2+c}}}{4\,a\,d^2}$$

Result(type 3, 353 leaves):

$$\frac{1}{16\,\sqrt{(d\,x^2+c)\,(a\,d\,x^2+a\,c+b)}\,a^2\,d^2\,\sqrt{a\,d^2}}\left(\sqrt{\frac{a\,d\,x^2+a\,c+b}{d\,x^2+c}}\,(d\,x^2+c)\left(4\,a\,\sqrt{x^4\,a\,d^2+2\,x^2\,a\,c\,d+b\,d\,x^2+c^2\,a+b\,c}\,x^2\,d\,\sqrt{a\,d^2}\right.\right.$$
$$+4\,b\ln\left(\frac{2\,a\,d^2\,x^2+2\,a\,c\,d+2\,\sqrt{x^4\,a\,d^2+2\,x^2\,a\,c\,d+b\,d\,x^2+c^2\,a+b\,c}\,\sqrt{a\,d^2}+b\,d}{2\,\sqrt{a\,d^2}}\right)a\,c\,d-4\,a\,\sqrt{x^4\,a\,d^2+2\,x^2\,a\,c\,d+b\,d\,x^2+c^2\,a+b\,c}\,c\,\sqrt{a\,d^2}$$
$$\left.\left.+3\ln\left(\frac{2\,a\,d^2\,x^2+2\,a\,c\,d+2\,\sqrt{x^4\,a\,d^2+2\,x^2\,a\,c\,d+b\,d\,x^2+c^2\,a+b\,c}\,\sqrt{a\,d^2}+b\,d}{2\,\sqrt{a\,d^2}}\right)b^2\,d-6\,\sqrt{x^4\,a\,d^2+2\,x^2\,a\,c\,d+b\,d\,x^2+c^2\,a+b\,c}\,b\,\sqrt{a\,d^2}\right)\right)$$

Problem 94: Result more than twice size of optimal antiderivative.

$$\int\frac{1}{x\,\sqrt{a+\frac{b}{d\,x^2+c}}}\,dx$$

Optimal(type 3, 80 leaves, 6 steps):

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{\frac{a\,d\,x^2+a\,c+b}{d\,x^2+c}}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}\,\sqrt{\frac{a\,d\,x^2+a\,c+b}{d\,x^2+c}}}{\sqrt{a\,c+b}}\right)\sqrt{c}}{\sqrt{a\,c+b}}$$

Result(type 3, 312 leaves):

$$-\frac{1}{2\,\sqrt{(d\,x^2+c)\,(a\,d\,x^2+a\,c+b)}\,(a\,c+b)\,\sqrt{a\,d^2}}\left(\sqrt{\frac{a\,d\,x^2+a\,c+b}{d\,x^2+c}}\,(d\,x^2+c)\left(\right.\right.$$
$$-\ln\left(\frac{2\,a\,d^2\,x^2+2\,a\,c\,d+2\,\sqrt{x^4\,a\,d^2+2\,x^2\,a\,c\,d+b\,d\,x^2+c^2\,a+b\,c}\,\sqrt{a\,d^2}+b\,d}{2\,\sqrt{a\,d^2}}\right)a\,c\,d$$
$$+\sqrt{c^2\,a+b\,c}\,\ln\left(\frac{2\,x^2\,a\,c\,d+b\,d\,x^2+2\,c^2\,a+2\,\sqrt{c^2\,a+b\,c}\,\sqrt{x^4\,a\,d^2+2\,x^2\,a\,c\,d+b\,d\,x^2+c^2\,a+b\,c}+2\,b\,c}{x^2}\right)\sqrt{a\,d^2}$$
$$\left.\left.-\ln\left(\frac{2\,a\,d^2\,x^2+2\,a\,c\,d+2\,\sqrt{x^4\,a\,d^2+2\,x^2\,a\,c\,d+b\,d\,x^2+c^2\,a+b\,c}\,\sqrt{a\,d^2}+b\,d}{2\,\sqrt{a\,d^2}}\right)b\,d\right)\right)$$

Problem 95: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^3 \sqrt{a + \frac{b}{d x^2 + c}}}\, dx$$

Optimal(type 3, 92 leaves, 5 steps):

$$-\frac{b\, d\, \operatorname{arctanh}\left(\frac{\sqrt{c}\, \sqrt{\frac{a d x^2 + a c + b}{d x^2 + c}}}{\sqrt{a c + b}}\right)}{2\, (a c + b)^{3/2} \sqrt{c}} - \frac{(d x^2 + c) \sqrt{\frac{a d x^2 + a c + b}{d x^2 + c}}}{2\, (a c + b)\, x^2}$$

Result(type 3, 451 leaves):

$$-\frac{1}{4 \sqrt{(d x^2 + c)\, (a d x^2 + a c + b)}\, (a c + b)^2 c x^2 \sqrt{c^2 a + b c}} \Bigg(\sqrt{\frac{a d x^2 + a c + b}{d x^2 + c}}\, (d x^2 + c) \Bigg($$

$$-2 a d^2 \sqrt{x^4 a d^2 + 2 x^2 a c d + b d x^2 + c^2 a + b c}\, x^4 \sqrt{c^2 a + b c}$$

$$+ \ln\left(\frac{2 x^2 a c d + b d x^2 + 2 c^2 a + 2 \sqrt{c^2 a + b c}\, \sqrt{x^4 a d^2 + 2 x^2 a c d + b d x^2 + c^2 a + b c} + 2 b c}{x^2}\right) x^2 a b c^2 d$$

$$- 4 \sqrt{x^4 a d^2 + 2 x^2 a c d + b d x^2 + c^2 a + b c}\, a c d x^2 \sqrt{c^2 a + b c}$$

$$+ \ln\left(\frac{2 x^2 a c d + b d x^2 + 2 c^2 a + 2 \sqrt{c^2 a + b c}\, \sqrt{x^4 a d^2 + 2 x^2 a c d + b d x^2 + c^2 a + b c} + 2 b c}{x^2}\right) x^2 b^2 c d$$

$$- 2 \sqrt{x^4 a d^2 + 2 x^2 a c d + b d x^2 + c^2 a + b c}\, b d x^2 \sqrt{c^2 a + b c} + 2 \left(x^4 a d^2 + 2 x^2 a c d + b d x^2 + c^2 a + b c\right)^{3/2} \sqrt{c^2 a + b c} \Bigg) \Bigg)$$

Problem 96: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^5 \sqrt{a + \frac{b}{d x^2 + c}}}\, dx$$

Optimal(type 3, 157 leaves, 6 steps):

$$\frac{b\, (4 a c + b)\, d^2 \operatorname{arctanh}\left(\frac{\sqrt{c}\, \sqrt{\frac{a d x^2 + a c + b}{d x^2 + c}}}{\sqrt{a c + b}}\right)}{8 c^{3/2} (a c + b)^{5/2}} + \frac{(4 a c + b)\, d\, (d x^2 + c) \sqrt{\frac{a d x^2 + a c + b}{d x^2 + c}}}{8 c\, (a c + b)^2 x^2} - \frac{(d x^2 + c)^2 \sqrt{\frac{a d x^2 + a c + b}{d x^2 + c}}}{4 c\, (a c + b)\, x^4}$$

Result(type 3, 921 leaves):

$$\frac{1}{16\sqrt{(d x^2+c)(a d x^2+a c+b)}\,(a c+b)^3 c^2 (c^2 a+b c)^{3/2} x^4}\Bigg(\sqrt{\frac{a d x^2+a c+b}{d x^2+c}}\,(d x^2+c)\Bigg($$

$$-12 a^2 d^3\sqrt{x^4 a d^2+2 x^2 a c d+b d x^2+c^2 a+b c}\,x^6 c\,(c^2 a+b c)^{3/2}$$

$$+4\ln\left(\frac{2 x^2 a c d+b d x^2+2 c^2 a+2\sqrt{c^2 a+b c}\,\sqrt{x^4 a d^2+2 x^2 a c d+b d x^2+c^2 a+b c}+2 b c}{x^2}\right) x^4 a^3 b c^5 d^2$$

$$-2 a d^3\sqrt{x^4 a d^2+2 x^2 a c d+b d x^2+c^2 a+b c}\,x^6 b\,(c^2 a+b c)^{3/2}$$

$$+9\ln\left(\frac{2 x^2 a c d+b d x^2+2 c^2 a+2\sqrt{c^2 a+b c}\,\sqrt{x^4 a d^2+2 x^2 a c d+b d x^2+c^2 a+b c}+2 b c}{x^2}\right) x^4 a^2 b^2 c^4 d^2$$

$$-20\sqrt{x^4 a d^2+2 x^2 a c d+b d x^2+c^2 a+b c}\,a^2 c^2 d^2 (c^2 a+b c)^{3/2} x^4$$

$$+6\ln\left(\frac{2 x^2 a c d+b d x^2+2 c^2 a+2\sqrt{c^2 a+b c}\,\sqrt{x^4 a d^2+2 x^2 a c d+b d x^2+c^2 a+b c}+2 b c}{x^2}\right) x^4 a b^3 c^3 d^2$$

$$-12 a d^2\sqrt{x^4 a d^2+2 x^2 a c d+b d x^2+c^2 a+b c}\,b c\,(c^2 a+b c)^{3/2} x^4$$

$$+\ln\left(\frac{2 x^2 a c d+b d x^2+2 c^2 a+2\sqrt{c^2 a+b c}\,\sqrt{x^4 a d^2+2 x^2 a c d+b d x^2+c^2 a+b c}+2 b c}{x^2}\right) x^4 b^4 c^2 d^2$$

$$-2\sqrt{x^4 a d^2+2 x^2 a c d+b d x^2+c^2 a+b c}\,b^2 d^2 (c^2 a+b c)^{3/2} x^4+12 d\,(x^4 a d^2+2 x^2 a c d+b d x^2+c^2 a+b c)^{3/2} a c\,(c^2 a+b c)^{3/2} x^2$$

$$+2 d\,(x^4 a d^2+2 x^2 a c d+b d x^2+c^2 a+b c)^{3/2} b\,(c^2 a+b c)^{3/2} x^2-4 (c^2 a+b c)^{3/2} (x^4 a d^2+2 x^2 a c d+b d x^2+c^2 a+b c)^{3/2} a c^2-4 (c^2 a + b c)^{3/2} (x^4 a d^2+2 x^2 a c d+b d x^2+c^2 a+b c)^{3/2} b c\Bigg)\Bigg)$$

Problem 100: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{\frac{a}{x^7}}}{\sqrt{x^5+1}}\,dx$$

Optimal(type 2, 17 leaves, 2 steps):

$$-\frac{2x\sqrt{\frac{a}{x^7}}\sqrt{x^5+1}}{5}$$

Result(type 2, 36 leaves):

$$-\frac{2(1+x)(x^4-x^3+x^2-x+1)x\sqrt{\frac{a}{x^7}}}{5\sqrt{x^5+1}}$$

Problem 105: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\frac{\sqrt{ax}}{\sqrt{x^3+1}}\,dx$$

Optimal(type 3, 15 leaves, 3 steps):

$$\frac{2\,\mathrm{arcsinh}\left(\frac{(ax)^{3/2}}{a^{3/2}}\right)\sqrt{a}}{3}$$

Result(type 4, 320 leaves):

$$-\left(4\sqrt{ax}\sqrt{x^3+1}\,a\left(I\sqrt{3}+1\right)\sqrt{\frac{\left(3+I\sqrt{3}\right)x}{\left(I\sqrt{3}+1\right)(1+x)}}\,(1+x)^2\sqrt{\frac{I\sqrt{3}+2x-1}{\left(I\sqrt{3}-1\right)(1+x)}}\sqrt{\frac{I\sqrt{3}-2x+1}{\left(I\sqrt{3}+1\right)(1+x)}}\left(\mathrm{EllipticF}\left(\sqrt{\frac{\left(3+I\sqrt{3}\right)x}{\left(I\sqrt{3}+1\right)(1+x)}},\sqrt{\frac{\left(-3+I\sqrt{3}\right)\left(I\sqrt{3}+1\right)}{\left(I\sqrt{3}-1\right)\left(3+I\sqrt{3}\right)}}\right)\right.\right.$$
$$\left.\left.-\mathrm{EllipticPi}\left(\sqrt{\frac{\left(3+I\sqrt{3}\right)x}{\left(I\sqrt{3}+1\right)(1+x)}},\frac{I\sqrt{3}+1}{3+I\sqrt{3}},\sqrt{\frac{\left(-3+I\sqrt{3}\right)\left(I\sqrt{3}+1\right)}{\left(I\sqrt{3}-1\right)\left(3+I\sqrt{3}\right)}}\right)\right)\right)\Big/\left(\sqrt{(x^3+1)ax}\left(3+I\sqrt{3}\right)\sqrt{-ax(1+x)\left(I\sqrt{3}+2x-1\right)\left(I\sqrt{3}-2x+1\right)}\right)$$

Problem 109: Result more than twice size of optimal antiderivative.

$$\int\frac{x}{\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^2}\,dx$$

Optimal(type 3, 137 leaves, 8 steps):

$$\frac{(a+c)x^2}{2(a-c)^2}+\frac{2bx^3}{3(a-c)^2}-\frac{2(bx+a)^{3/2}(bx+c)^{3/2}}{3b^2(a-c)^2}-\frac{(a+c)\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{bx+c}}\right)}{4b^2}+\frac{(a+c)(bx+a)^{3/2}\sqrt{bx+c}}{2b^2(a-c)^2}$$

$$-\frac{(a+c)\sqrt{bx+a}\,\sqrt{bx+c}}{4b^2(a-c)}$$

Result(type 3, 430 leaves):

$$\frac{x^2 a}{2(a-c)^2}+\frac{x^2 c}{2(a-c)^2}+\frac{2bx^3}{3(a-c)^2}-\frac{1}{24(a-c)^2 b^2\sqrt{b^2x^2+abx+bcx+ac}}\Bigg(\sqrt{bx+a}\,\sqrt{bx+c}\,\Bigg(16\,\mathrm{csgn}(b)\,x^2 b^2\sqrt{b^2x^2+abx+bcx+ac}$$

$$+4\sqrt{b^2x^2+abx+bcx+ac}\,\mathrm{csgn}(b)\,xab+4\sqrt{b^2x^2+abx+bcx+ac}\,\mathrm{csgn}(b)\,xbc-6\sqrt{b^2x^2+abx+bcx+ac}\,\mathrm{csgn}(b)\,a^2$$

$$+4\sqrt{b^2x^2+abx+bcx+ac}\,\mathrm{csgn}(b)\,ac-6\sqrt{b^2x^2+abx+bcx+ac}\,\mathrm{csgn}(b)\,c^2$$

$$+3\ln\left(\frac{\left(2\sqrt{b^2x^2+abx+bcx+ac}\,\mathrm{csgn}(b)+2bx+a+c\right)\mathrm{csgn}(b)}{2}\right)a^3$$

$$-3\ln\left(\frac{\left(2\sqrt{b^2x^2+abx+bcx+ac}\,\mathrm{csgn}(b)+2bx+a+c\right)\mathrm{csgn}(b)}{2}\right)a^2 c$$

$$-3\ln\left(\frac{\left(2\sqrt{b^2x^2+abx+bcx+ac}\,\mathrm{csgn}(b)+2bx+a+c\right)\mathrm{csgn}(b)}{2}\right)a c^2$$

$$+3\ln\left(\frac{\left(2\sqrt{b^2x^2+abx+bcx+ac}\,\mathrm{csgn}(b)+2bx+a+c\right)\mathrm{csgn}(b)}{2}\right)c^3\Bigg)\mathrm{csgn}(b)\Bigg)$$

Problem 112: Result more than twice size of optimal antiderivative.

$$\int\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx$$

Optimal(type 3, 17 leaves, 4 steps):

$$2x+\arcsin(x)+x\sqrt{-x^2+1}$$

Result(type 3, 57 leaves):

$$2x-\sqrt{1+x}\,(1-x)^{3/2}+\sqrt{1+x}\,\sqrt{1-x}+\frac{\sqrt{(1-x)(1+x)}\,\arcsin(x)}{\sqrt{1-x}\,\sqrt{1+x}}$$

Problem 115: Result more than twice size of optimal antiderivative.

$$\int\frac{x^3}{\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^2}\,dx$$

Optimal(type 3, 161 leaves, 8 steps):

$$\frac{a x^2}{(b-c)^2}+\frac{(b+c) x^3}{3 (b-c)^2}-\frac{2 (b x+a)^{3/2} (c x+a)^{3/2}}{3 b (b-c)^2 c}-\frac{a^3 (b+c) \operatorname{arctanh}\left(\frac{\sqrt{c}\,\sqrt{b x+a}}{\sqrt{b}\,\sqrt{c x+a}}\right)}{4 b^{5/2} c^{5/2}}+\frac{a (b+c) (b x+a)^{3/2} \sqrt{c x+a}}{2 b^2 (b-c)^2 c}$$
$$+\frac{a^2 (b+c) \sqrt{b x+a}\,\sqrt{c x+a}}{4 b^2 (b-c) c^2}$$

Result(type 3, 516 leaves):

$$\frac{x^3 b}{3 (b-c)^2}+\frac{x^3 c}{3 (b-c)^2}+\frac{a x^2}{(b-c)^2}$$
$$-\frac{1}{24 (b-c)^2 \sqrt{b c x^2+a b x+a c x+a^2}\; b^2 c^2 \sqrt{b c}}\Bigg(\sqrt{b x+a}\,\sqrt{c x+a}\,\Bigg(16 x^2 b^2 c^2 \sqrt{b c}\,\sqrt{b c x^2+a b x+a c x+a^2}$$
$$+3 \ln\left(\frac{2 b c x+2 \sqrt{b c x^2+a b x+a c x+a^2}\,\sqrt{b c}+a b+a c}{2 \sqrt{b c}}\right) a^3 b^3-3 \ln\left(\frac{2 b c x+2 \sqrt{b c x^2+a b x+a c x+a^2}\,\sqrt{b c}+a b+a c}{2 \sqrt{b c}}\right) a^3 b^2 c$$
$$-3 \ln\left(\frac{2 b c x+2 \sqrt{b c x^2+a b x+a c x+a^2}\,\sqrt{b c}+a b+a c}{2 \sqrt{b c}}\right) a^3 b c^2+3 \ln\left(\frac{2 b c x+2 \sqrt{b c x^2+a b x+a c x+a^2}\,\sqrt{b c}+a b+a c}{2 \sqrt{b c}}\right) a^3 c^3$$
$$+4 \sqrt{b c}\,\sqrt{b c x^2+a b x+a c x+a^2}\; x a b^2 c+4 \sqrt{b c}\,\sqrt{b c x^2+a b x+a c x+a^2}\; x a b c^2-6 \sqrt{b c}\,\sqrt{b c x^2+a b x+a c x+a^2}\; a^2 b^2$$
$$+4 \sqrt{b c}\,\sqrt{b c x^2+a b x+a c x+a^2}\; a^2 b c-6 \sqrt{b c}\,\sqrt{b c x^2+a b x+a c x+a^2}\; a^2 c^2\Bigg)\Bigg)$$

Problem 116: Result more than twice size of optimal antiderivative.

$$\int \frac{x}{\left(\sqrt{b x+a}+\sqrt{c x+a}\right)^2}\, dx$$

Optimal(type 3, 115 leaves, 9 steps):

$$\frac{(b+c) x}{(b-c)^2}+\frac{4 a \operatorname{arctanh}\left(\frac{\sqrt{b x+a}}{\sqrt{c x+a}}\right)}{(b-c)^2}+\frac{2 a \ln(x)}{(b-c)^2}-\frac{2 a (b+c) \operatorname{arctanh}\left(\frac{\sqrt{c}\,\sqrt{b x+a}}{\sqrt{b}\,\sqrt{c x+a}}\right)}{(b-c)^2 \sqrt{b}\,\sqrt{c}}-\frac{2 \sqrt{b x+a}\,\sqrt{c x+a}}{(b-c)^2}$$

Result(type 3, 265 leaves):

$$\frac{b x}{(b-c)^2}+\frac{c x}{(b-c)^2}+\frac{2 a \ln(x)}{(b-c)^2}$$
$$-\frac{1}{(b-c)^2 \sqrt{b c x^2+a b x+a c x+a^2}\,\sqrt{b c}}\Bigg(\sqrt{b x+a}\,\sqrt{c x+a}\,\Bigg(\ln\left(\frac{2 b c x+2 \sqrt{b c x^2+a b x+a c x+a^2}\,\sqrt{b c}+a b+a c}{2 \sqrt{b c}}\right) \operatorname{csgn}(a)\, a b$$

$$+\ln\left(\frac{2bcx+2\sqrt{bcx^2+abx+acx+a^2}\,\sqrt{bc}+ab+ac}{2\sqrt{bc}}\right)\operatorname{csgn}(a)\,a\,c+2\sqrt{bc}\,\sqrt{bcx^2+abx+acx+a^2}\,\operatorname{csgn}(a)$$
$$-2\sqrt{bc}\,\ln\left(\frac{a\left(2\sqrt{bcx^2+abx+acx+a^2}\,\operatorname{csgn}(a)+bx+cx+2a\right)}{x}\right)a\Bigg)\operatorname{csgn}(a)\Bigg)$$

Problem 117: Result more than twice size of optimal antiderivative.

$$\int\frac{1}{x\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^2}\,dx$$

Optimal(type 3, 110 leaves, 6 steps):

$$-\frac{a}{(b-c)^2x^2}+\frac{-b-c}{(b-c)^2x}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{cx+a}}\right)}{2a}+\frac{(cx+a)^{3/2}\sqrt{bx+a}}{a(b-c)^2x^2}+\frac{\sqrt{bx+a}\,\sqrt{cx+a}}{2a(b-c)x}$$

Result(type 3, 312 leaves):

$$-\frac{b}{(b-c)^2x}-\frac{c}{(b-c)^2x}-\frac{a}{(b-c)^2x^2}+\frac{1}{4(b-c)^2a\sqrt{bcx^2+abx+acx+a^2}\,x^2}\Bigg(\sqrt{bx+a}\,\sqrt{cx+a}\,\Bigg($$
$$-\ln\left(\frac{a\left(2\sqrt{bcx^2+abx+acx+a^2}\,\operatorname{csgn}(a)+bx+cx+2a\right)}{x}\right)x^2b^2+2\ln\left(\frac{a\left(2\sqrt{bcx^2+abx+acx+a^2}\,\operatorname{csgn}(a)+bx+cx+2a\right)}{x}\right)x^2bc$$
$$-\ln\left(\frac{a\left(2\sqrt{bcx^2+abx+acx+a^2}\,\operatorname{csgn}(a)+bx+cx+2a\right)}{x}\right)x^2c^2+2\sqrt{bcx^2+abx+acx+a^2}\,\operatorname{csgn}(a)\,x\,b$$
$$+2\sqrt{bcx^2+abx+acx+a^2}\,\operatorname{csgn}(a)\,x\,c+4\operatorname{csgn}(a)\,a\sqrt{bcx^2+abx+acx+a^2}\Bigg)\operatorname{csgn}(a)\Bigg)$$

Problem 118: Result more than twice size of optimal antiderivative.

$$\int\frac{1}{x^2\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^2}\,dx$$

Optimal(type 3, 150 leaves, 7 steps):

$$-\frac{2a}{3(b-c)^2x^3}+\frac{-b-c}{2(b-c)^2x^2}+\frac{2(bx+a)^{3/2}(cx+a)^{3/2}}{3a^2(b-c)^2x^3}+\frac{(b+c)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{cx+a}}\right)}{4a^2}-\frac{(b+c)(cx+a)^{3/2}\sqrt{bx+a}}{2a^2(b-c)^2x^2}$$
$$-\frac{(b+c)\sqrt{bx+a}\,\sqrt{cx+a}}{4a^2(b-c)x}$$

Result(type 3, 456 leaves):

$$-\frac{b}{2 x^2 (b-c)^2}-\frac{c}{2 x^2 (b-c)^2}-\frac{2 a}{3 (b-c)^2 x^3}-\frac{1}{24 (b-c)^2 a^2 \sqrt{b c x^2+a b x+a c x+a^2}\, x^3}\Bigg(\sqrt{b x+a}\,\sqrt{c x+a}\,\Bigg($$

$$-3 \ln\left(\frac{a \left(2 \sqrt{b c x^2+a b x+a c x+a^2}\,\mathrm{csgn}(a)+b x+c x+2 a\right)}{x}\right) x^3 b^3+3 \ln\left(\frac{a \left(2 \sqrt{b c x^2+a b x+a c x+a^2}\,\mathrm{csgn}(a)+b x+c x+2 a\right)}{x}\right) x^3 b^2 c$$

$$+3 \ln\left(\frac{a \left(2 \sqrt{b c x^2+a b x+a c x+a^2}\,\mathrm{csgn}(a)+b x+c x+2 a\right)}{x}\right) x^3 b c^2-3 \ln\left(\frac{a \left(2 \sqrt{b c x^2+a b x+a c x+a^2}\,\mathrm{csgn}(a)+b x+c x+2 a\right)}{x}\right) x^3 c^3$$

$$+6 \sqrt{b c x^2+a b x+a c x+a^2}\,\mathrm{csgn}(a)\, x^2 b^2-4 \sqrt{b c x^2+a b x+a c x+a^2}\,\mathrm{csgn}(a)\, x^2 b c+6 \sqrt{b c x^2+a b x+a c x+a^2}\,\mathrm{csgn}(a)\, x^2 c^2$$

$$-4\,\mathrm{csgn}(a)\, a \sqrt{b c x^2+a b x+a c x+a^2}\, x b-4\,\mathrm{csgn}(a)\, a \sqrt{b c x^2+a b x+a c x+a^2}\, x c-16 \sqrt{b c x^2+a b x+a c x+a^2}\, a^2\,\mathrm{csgn}(a)\Bigg)\mathrm{csgn}(a)\Bigg)$$

Problem 121: Result more than twice size of optimal antiderivative.

$$\int \left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right) dx$$

Optimal(type 3, 20 leaves, 5 steps):

$$-2 x-\arcsin(x)-x\sqrt{-x^2+1}$$

Result(type 3, 58 leaves):

$$-2 x+\sqrt{1+x}\,(1-x)^{3/2}-\sqrt{1+x}\,\sqrt{1-x}-\frac{\sqrt{(1-x)(1+x)}\,\arcsin(x)}{\sqrt{1-x}\,\sqrt{1+x}}$$

Problem 122: Result more than twice size of optimal antiderivative.

$$\int \frac{-\sqrt{-1+x}+\sqrt{1+x}}{\sqrt{-1+x}+\sqrt{1+x}}\, dx$$

Optimal(type 3, 23 leaves, 9 steps):

$$\frac{x^2}{2}+\frac{\operatorname{arccosh}(x)}{2}-\frac{x\sqrt{-1+x}\,\sqrt{1+x}}{2}$$

Result(type 3, 61 leaves):

$$\frac{x^2}{2}-\frac{\sqrt{-1+x}\,(1+x)^{3/2}}{2}+\frac{\sqrt{-1+x}\,\sqrt{1+x}}{2}+\frac{\sqrt{(-1+x)(1+x)}\,\ln\left(x+\sqrt{x^2-1}\right)}{2\sqrt{1+x}\,\sqrt{-1+x}}$$

Problem 123: Unable to integrate problem.

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^n dx$$

Optimal(type 5, 113 leaves, 4 steps):

$$\frac{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{1+n}}{2e(1+n)}+\frac{af^2\,\mathrm{hypergeom}\left([2,1+n],[n+2],\frac{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}{d}\right)\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{1+n}}{2d^2e(1+n)}$$

Result(type 8, 25 leaves):

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^n dx$$

Problem 124: Unable to integrate problem.

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{5/2} dx$$

Optimal(type 3, 191 leaves, 6 steps):

$$-\frac{5ad^{3/2}f^2\,\mathrm{arctanh}\left(\frac{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{\sqrt{d}}\right)}{2e}+\frac{af^2\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}}{3e}+\frac{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{7/2}}{7e}$$
$$+\frac{2adf^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{e}-\frac{ad^2f^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}$$

Result(type 8, 25 leaves):

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{5/2} dx$$

Problem 125: Unable to integrate problem.

$$\int \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\, dx$$

Optimal(type 3, 123 leaves, 6 steps):

$$-\frac{a f^2 \operatorname{arctanh}\left(\frac{\sqrt{d+e x+f \sqrt{a+\frac{e^2 x^2}{f^2}}}}{\sqrt{d}}\right)}{2 e \sqrt{d}}+\frac{\left(d+e x+f \sqrt{a+\frac{e^2 x^2}{f^2}}\right)^{3/2}}{3 e}-\frac{a f^2 \sqrt{d+e x+f \sqrt{a+\frac{e^2 x^2}{f^2}}}}{2 e \left(f \sqrt{a+\frac{e^2 x^2}{f^2}}+e x\right)}$$

Result(type 8, 25 leaves):

$$\int \sqrt{d+e x+f \sqrt{a+\frac{e^2 x^2}{f^2}}}\, dx$$

Problem 126: Unable to integrate problem.

$$\int \frac{1}{\sqrt{d+e x+f \sqrt{a+\frac{e^2 x^2}{f^2}}}}\, dx$$

Optimal(type 3, 125 leaves, 5 steps):

$$\frac{a f^2 \operatorname{arctanh}\left(\frac{\sqrt{d+e x+f \sqrt{a+\frac{e^2 x^2}{f^2}}}}{\sqrt{d}}\right)}{2 d^{3/2} e}+\frac{\sqrt{d+e x+f \sqrt{a+\frac{e^2 x^2}{f^2}}}}{e}-\frac{a f^2 \sqrt{d+e x+f \sqrt{a+\frac{e^2 x^2}{f^2}}}}{2 d e \left(f \sqrt{a+\frac{e^2 x^2}{f^2}}+e x\right)}$$

Result(type 8, 25 leaves):

$$\int \frac{1}{\sqrt{d+e x+f \sqrt{a+\frac{e^2 x^2}{f^2}}}}\, dx$$

Problem 127: Unable to integrate problem.

$$\int \sqrt{a x+b \sqrt{c+\frac{a^2 x^2}{b^2}}}\, dx$$

Optimal(type 2, 59 leaves, 3 steps):

$$\frac{\left(a x+b \sqrt{c+\frac{a^2 x^2}{b^2}}\right)^{3/2}}{3 a}-\frac{b^2 c}{a \sqrt{a x+b \sqrt{c+\frac{a^2 x^2}{b^2}}}}$$

Result(type 8, 24 leaves):

$$\int \sqrt{a x+b \sqrt{c+\frac{a^2 x^2}{b^2}}}\, dx$$

Problem 128: Result unnecessarily involves higher level functions.

$$\int \sqrt{1+\sqrt{-x^2+1}}\, dx$$

Optimal(type 2, 35 leaves, 1 step):

$$-\frac{2 x^3}{3 \left(1+\sqrt{-x^2+1}\right)^{3/2}}+\frac{2 x}{\sqrt{1+\sqrt{-x^2+1}}}$$

Result(type 3, 59 leaves):

$$\frac{\frac{I}{8}\left(\frac{32 I \sqrt{\pi} \sqrt{2}\, x^3 \cos\left(\frac{3 \arcsin(x)}{2}\right)}{3}-\frac{8 I \sqrt{\pi} \sqrt{2} \left(-\frac{4}{3} x^4+\frac{2}{3} x^2+\frac{2}{3}\right) \sin\left(\frac{3 \arcsin(x)}{2}\right)}{\sqrt{-x^2+1}}\right)}{\sqrt{\pi}}$$

Problem 129: Unable to integrate problem.

$$\int \sqrt{a+b \sqrt{\frac{a^2}{b^2}+c x^2}}\, dx$$

Optimal(type 2, 56 leaves, 1 step):

$$\frac{2 b^2 c x^3}{3 \left(a+b \sqrt{\frac{a^2}{b^2}+c x^2}\right)^{3/2}}+\frac{2 a x}{\sqrt{a+b \sqrt{\frac{a^2}{b^2}+c x^2}}}$$

Result(type 8, 23 leaves):

$$\int \sqrt{a+b \sqrt{\frac{a^2}{b^2}+c x^2}}\, dx$$

Problem 130: Unable to integrate problem.

$$\int\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^n dx$$

Optimal(type 5, 158 leaves, 4 steps):

$$\frac{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{1+n}}{2e(1+n)}+\frac{f^2(-b^2f^2+4ae^2)\,\mathrm{hypergeom}\left([2,1+n],[n+2],\frac{2e\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)}{-bf^2+2de}\right)\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{1+n}}{2e(-bf^2+2de)^2(1+n)}$$

Result(type 8, 28 leaves):

$$\int\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^n dx$$

Problem 132: Result more than twice size of optimal antiderivative.

$$\int\frac{1}{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\,dx$$

Optimal(type 3, 205 leaves, 3 steps):

$$\frac{2(aef^2-bdf^2+d^2e)\ln\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)}{(-bf^2+2de)^2}-\frac{f^2(-b^2f^2+4ae^2)\ln\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)}{2e(-bf^2+2de)^2}-\frac{f^2(-b^2f^2+4ae^2)}{2e(-bf^2+2de)\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)}$$

Result(type ?, 4917 leaves): Display of huge result suppressed!

Problem 133: Unable to integrate problem.

$$\int\frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{5/2}}\,dx$$

Optimal(type 3, 303 leaves, 6 steps):

$$\frac{5 f^2 \left(-b^2 f^2+4 a e^2\right) \operatorname{arctanh}\left(\frac{\sqrt{2}\, \sqrt{e}\, \sqrt{d+e x+f \sqrt{a+b x+\frac{e^2 x^2}{f^2}}}}{\sqrt{-b f^2+2 d e}}\right) \sqrt{2}\, \sqrt{e}}{\left(-b f^2+2 d e\right)^{7/2}}-\frac{4 \left(a e f^2-b d f^2+d^2 e\right)}{3 \left(-b f^2+2 d e\right)^2 \left(d+e x+f \sqrt{a+b x+\frac{e^2 x^2}{f^2}}\right)^{3/2}}$$

$$-\frac{4 f^2 \left(-b^2 f^2+4 a e^2\right)}{\left(-b f^2+2 d e\right)^3 \sqrt{d+e x+f \sqrt{a+b x+\frac{e^2 x^2}{f^2}}}}-\frac{2 e f^2 \left(-b^2 f^2+4 a e^2\right) \sqrt{d+e x+f \sqrt{a+b x+\frac{e^2 x^2}{f^2}}}}{\left(-b f^2+2 d e\right)^3 \left(b f^2+2 e \left(e x+f \sqrt{a+\frac{x \left(b f^2+e^2 x\right)}{f^2}}\right)\right)}$$

Result(type 8, 28 leaves):

$$\int \frac{1}{\left(d+e x+f \sqrt{a+b x+\frac{e^2 x^2}{f^2}}\right)^{5/2}}\, dx$$

Problem 134: Unable to integrate problem.

$$\int \left(x^2+a\right) \left(x-\sqrt{x^2+a}\right)^n dx$$

Optimal(type 3, 100 leaves, 3 steps):

$$-\frac{a^3 \left(x-\sqrt{x^2+a}\right)^{-3+n}}{8 \left(3-n\right)}-\frac{3 a^2 \left(x-\sqrt{x^2+a}\right)^{-1+n}}{8 \left(1-n\right)}+\frac{3 a \left(x-\sqrt{x^2+a}\right)^{1+n}}{8 \left(1+n\right)}+\frac{\left(x-\sqrt{x^2+a}\right)^{3+n}}{8 \left(3+n\right)}$$

Result(type 8, 21 leaves):

$$\int \left(x^2+a\right) \left(x-\sqrt{x^2+a}\right)^n dx$$

Problem 135: Unable to integrate problem.

$$\int \frac{\left(x-\sqrt{x^2+a}\right)^n}{x^2+a}\, dx$$

Optimal(type 5, 57 leaves, 2 steps):

$$\frac{2 \operatorname{hypergeom}\left(\left[1, \frac{1}{2}+\frac{n}{2}\right], \left[\frac{3}{2}+\frac{n}{2}\right], -\frac{\left(x-\sqrt{x^2+a}\right)^2}{a}\right) \left(x-\sqrt{x^2+a}\right)^{1+n}}{a \left(1+n\right)}$$

Result(type 8, 23 leaves):

$$\int \frac{\left(x-\sqrt{x^2+a}\right)^n}{x^2+a}\,dx$$

Problem 136: Unable to integrate problem.

$$\int \frac{\left(x-\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^2}\,dx$$

Optimal(type 5, 57 leaves, 2 steps):

$$\frac{8\,\mathrm{hypergeom}\left(\left[3,\frac{3}{2}+\frac{n}{2}\right],\left[\frac{5}{2}+\frac{n}{2}\right],-\frac{\left(x-\sqrt{x^2+a}\right)^2}{a}\right)\left(x-\sqrt{x^2+a}\right)^{3+n}}{a^3\,(3+n)}$$

Result(type 8, 23 leaves):

$$\int \frac{\left(x-\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^2}\,dx$$

Problem 137: Unable to integrate problem.

$$\int \frac{\left(x-\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^{3/2}}\,dx$$

Optimal(type 5, 57 leaves, 2 steps):

$$-\frac{4\,\mathrm{hypergeom}\left(\left[2,\frac{n}{2}+1\right],\left[2+\frac{n}{2}\right],-\frac{\left(x-\sqrt{x^2+a}\right)^2}{a}\right)\left(x-\sqrt{x^2+a}\right)^{n+2}}{a^2\,(n+2)}$$

Result(type 8, 23 leaves):

$$\int \frac{\left(x-\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^{3/2}}\,dx$$

Problem 138: Unable to integrate problem.

$$\int \left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$

Optimal(type 3, 99 leaves, 4 steps):

$$\frac{\left(-a f^{2}+d^{2}\right)\left(d+e x+f\sqrt{a+\frac{2 d e x}{f^{2}}+\frac{e^{2} x^{2}}{f^{2}}}\right)^{-1+n}}{2 e (1-n)}+\frac{\left(d+e x+f\sqrt{a+\frac{2 d e x}{f^{2}}+\frac{e^{2} x^{2}}{f^{2}}}\right)^{1+n}}{2 e (1+n)}$$

Result(type 8, 33 leaves):

$$\int\left(d+e x+f\sqrt{a+\frac{2 d e x}{f^{2}}+\frac{e^{2} x^{2}}{f^{2}}}\right)^{n} dx$$

Problem 139: Unable to integrate problem.

$$\int\frac{\left(d+e x+f\sqrt{a+\frac{2 d e x}{f^{2}}+\frac{e^{2} x^{2}}{f^{2}}}\right)^{n}}{\sqrt{a+\frac{2 d e x}{f^{2}}+\frac{e^{2} x^{2}}{f^{2}}}} dx$$

Optimal(type 3, 39 leaves, 3 steps):

$$\frac{f\left(d+e x+f\sqrt{a+\frac{2 d e x}{f^{2}}+\frac{e^{2} x^{2}}{f^{2}}}\right)^{n}}{e n}$$

Result(type 8, 56 leaves):

$$\int\frac{\left(d+e x+f\sqrt{a+\frac{2 d e x}{f^{2}}+\frac{e^{2} x^{2}}{f^{2}}}\right)^{n}}{\sqrt{a+\frac{2 d e x}{f^{2}}+\frac{e^{2} x^{2}}{f^{2}}}} dx$$

Problem 140: Result more than twice size of optimal antiderivative.

$$\int\frac{1}{(b x+a)\sqrt{d x^{2}+c}\,\sqrt{f x^{2}+e}} dx$$

Optimal(type 4, 164 leaves, 7 steps):

$$-\frac{b\,\mathrm{arctanh}\left(\frac{\sqrt{a^{2} f+e b^{2}}\,\sqrt{d x^{2}+c}}{\sqrt{a^{2} d+b^{2} c}\,\sqrt{f x^{2}+e}}\right)}{\sqrt{a^{2} d+b^{2} c}\,\sqrt{a^{2} f+e b^{2}}}+\frac{\mathrm{EllipticPi}\left(\frac{x\sqrt{d}}{\sqrt{-c}},-\frac{b^{2} c}{a^{2} d},\sqrt{\frac{c f}{d e}}\right)\sqrt{-c}\,\sqrt{1+\frac{d x^{2}}{c}}\,\sqrt{1+\frac{f x^{2}}{e}}}{a\sqrt{d}\,\sqrt{d x^{2}+c}\,\sqrt{f x^{2}+e}}$$

Result(type 4, 352 leaves):

$$\frac{1}{2 a \sqrt{-\frac{d}{c}}\, \sqrt{\frac{a^4 d f+a^2 b^2 c f+a^2 b^2 d e+e c b^4}{b^4}}\, b \left(d f x^4+c f x^2+e x^2 d+e c\right)} \left(\left(2 \sqrt{\frac{d x^2+c}{c}}\, \sqrt{\frac{f x^2+e}{e}}\, \sqrt{\frac{a^4 d f+a^2 b^2 c f+a^2 b^2 d e+e c b^4}{b^4}}\right.\right.$$

$$\mathrm{EllipticPi}\left(\sqrt{-\frac{d}{c}}\, x, -\frac{b^2 c}{a^2 d}, \frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right) b$$

$$\left.-\operatorname{arctanh}\left(\frac{2 a^2 d f x^2+b^2 c f x^2+b^2 d e x^2+c f a^2+a^2 d e+2 b^2 e c}{2 b^2 \sqrt{\frac{a^4 d f+a^2 b^2 c f+a^2 b^2 d e+e c b^4}{b^4}}\, \sqrt{d f x^4+c f x^2+e x^2 d+e c}}\right) \sqrt{d f x^4+c f x^2+e x^2 d+e c}\, \sqrt{-\frac{d}{c}}\, a\right) \sqrt{f x^2+e}\, \sqrt{d x^2+c}\Bigg)$$

Problem 141: Result more than twice size of optimal antiderivative.

$$\int \frac{e-2 f(-1+n) x^n}{e^2+4 d f x^2+4 e f x^n+4 f^2 x^{2 n}}\, dx$$

Optimal(type 3, 28 leaves, 2 steps):

$$\frac{\arctan\left(\frac{2 x \sqrt{d}\, \sqrt{f}}{e+2 f x^n}\right)}{2 \sqrt{d}\, \sqrt{f}}$$

Result(type 3, 77 leaves):

$$-\frac{\ln\left(x^n+\frac{2 d f x+e \sqrt{-d f}}{2 \sqrt{-d f}\, f}\right)}{4 \sqrt{-d f}}+\frac{\ln\left(x^n+\frac{-2 d f x+e \sqrt{-d f}}{2 \sqrt{-d f}\, f}\right)}{4 \sqrt{-d f}}$$

Problem 143: Result is not expressed in closed-form.

$$\int \frac{x\left(-2 f x^3+2 e\right)}{4 f^2 x^6+4 d f x^4+4 e f x^3+e^2}\, dx$$

Optimal(type 3, 30 leaves, 2 steps):

$$\frac{\arctan\left(\frac{2 x^2 \sqrt{d}\, \sqrt{f}}{2 f x^3+e}\right)}{2 \sqrt{d}\, \sqrt{f}}$$

Result(type 7, 73 leaves):

$$-\frac{\displaystyle\sum_{\_R=RootOf(4f^2\_Z^6+4df\_Z^4+4ef\_Z^3+e^2)} \frac{(\_R^4 f-\_R e)\ln(x-\_R)}{6f\_R^5+4d\_R^3+3e\_R^2}}{2f}$$

Problem 144: Result more than twice size of optimal antiderivative.

$$\int \frac{x^m\left(e(1+m)+2f(-2+m)x^3\right)}{e^2+4efx^3+4f^2x^6+4dfx^{2+2m}}\,dx$$

Optimal(type 3, 32 leaves, 2 steps):

$$\frac{\arctan\left(\frac{2x^{1+m}\sqrt{d}\sqrt{f}}{2fx^3+e}\right)}{2\sqrt{d}\sqrt{f}}$$

Result(type 3, 77 leaves):

$$-\frac{\ln\left(x^m+\frac{(2fx^3+e)\sqrt{-df}}{2dfx}\right)}{4\sqrt{-df}}+\frac{\ln\left(x^m-\frac{(2fx^3+e)\sqrt{-df}}{2dfx}\right)}{4\sqrt{-df}}$$

Problem 145: Result more than twice size of optimal antiderivative.

$$\int \frac{x^m\left(e(1+m)+2f(-2+m)x^3\right)}{e^2+4efx^3+4f^2x^6-4dfx^{2+2m}}\,dx$$

Optimal(type 3, 32 leaves, 2 steps):

$$\frac{\operatorname{arctanh}\left(\frac{2x^{1+m}\sqrt{d}\sqrt{f}}{2fx^3+e}\right)}{2\sqrt{d}\sqrt{f}}$$

Result(type 3, 73 leaves):

$$\frac{\ln\left(x^m+\frac{(2fx^3+e)\sqrt{df}}{2dfx}\right)}{4\sqrt{df}}-\frac{\ln\left(x^m-\frac{(2fx^3+e)\sqrt{df}}{2dfx}\right)}{4\sqrt{df}}$$

Problem 146: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x\left(ac+bcx^2+d\sqrt{bx^2+a}\right)}\,dx$$

Optimal(type 3, 80 leaves, 7 steps):

$$\frac{c\ln(x)}{c^2 a-d^2}-\frac{c\ln\left(d+c\sqrt{b x^2+a}\right)}{c^2 a-d^2}+\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b x^2+a}}{\sqrt{a}}\right)}{(c^2 a-d^2)\sqrt{a}}$$

Result(type ?, 2174 leaves): Display of huge result suppressed!

Problem 147: Result is not expressed in closed-form.

$$\int\frac{x^5}{a c+b c x^3+d\sqrt{b x^3+a}}\,dx$$

Optimal(type 3, 63 leaves, 4 steps):

$$\frac{x^3}{3 b c}-\frac{2(c^2 a-d^2)\ln\left(d+c\sqrt{b x^3+a}\right)}{3 b^2 c^3}-\frac{2 d\sqrt{b x^3+a}}{3 c^2 b^2}$$

Result(type 7, 942 leaves):

$$-\frac{2 d\sqrt{b x^3+a}}{3 c^2 b^2}+\frac{1}{3 d b^4}\left(\mathrm{I}\sqrt{2}\left(\sum_{\_\alpha=RootOf(b c^2\_Z^3+c^2 a-d^2)}\frac{1}{\sqrt{b x^3+a}}\left((-a b^2)^{1/}\right.\right.\right.$$

$$\sqrt[3]{\frac{\frac{\mathrm{I}}{2} b\left(2x+\frac{-\mathrm{I}\sqrt{3}(-a b^2)^{1/3}+(-a b^2)^{1/3}}{b}\right)}{(-a b^2)^{1/3}}}\sqrt{\frac{b\left(x-\frac{(-a b^2)^{1/3}}{b}\right)}{-3(-a b^2)^{1/3}+\mathrm{I}\sqrt{3}(-a b^2)^{1/3}}}\sqrt{\frac{-\frac{\mathrm{I}}{2} b\left(2x+\frac{(-a b^2)^{1/3}+\mathrm{I}\sqrt{3}(-a b^2)^{1/3}}{b}\right)}{(-a b^2)^{1/3}}}\left(\mathrm{I}\right.$$

$$(-a b^2)^{1/3}\sqrt{3}\_\alpha b-\mathrm{I}(-a b^2)^{2/3}\sqrt{3}+2\_\alpha^2 b^2-(-a b^2)^{1/3}\_\alpha b-(-a b^2)^{2/3})$$

$$\operatorname{EllipticPi}\left(\frac{\sqrt{3}\sqrt{\frac{\mathrm{I}\left(x+\frac{(-a b^2)^{1/3}}{2b}-\frac{\mathrm{I}\sqrt{3}(-a b^2)^{1/3}}{2b}\right)\sqrt{3}\,b}{(-a b^2)^{1/3}}}}{3},\right.$$

$$-\frac{c^2\left(2\,I\sqrt{3}\,(-a\,b^2)^{1/3}\,\_\alpha^2\,b-I\sqrt{3}\,(-a\,b^2)^{2/3}\,\_\alpha+I\sqrt{3}\,a\,b-3\,(-a\,b^2)^{2/3}\,\_\alpha-3\,a\,b\right)}{2\,b\,d^2},\ \sqrt{\frac{I\sqrt{3}\,(-a\,b^2)^{1/3}}{b\left(-\frac{3\,(-a\,b^2)^{1/3}}{2\,b}+\frac{I\sqrt{3}\,(-a\,b^2)^{1/3}}{2\,b}\right)}}\Bigg)\Bigg)\Bigg)$$

$$a\Bigg)-\frac{1}{3\,b^4\,c^2}\Bigg(I\,d\sqrt{2}\Bigg(\sum_{\_\alpha=RootOf(b\,c^2\,\_Z^3+c^2\,a-d^2)}\frac{1}{\sqrt{b\,x^3+a}}\Bigg((-a\,b^2)^{1/}$$

$$3\sqrt{\frac{\frac{I}{2}\,b\left(2\,x+\frac{-I\sqrt{3}\,(-a\,b^2)^{1/3}+(-a\,b^2)^{1/3}}{b}\right)}{(-a\,b^2)^{1/3}}}\sqrt{\frac{b\left(x-\frac{(-a\,b^2)^{1/3}}{b}\right)}{-3\,(-a\,b^2)^{1/3}+I\sqrt{3}\,(-a\,b^2)^{1/3}}}\sqrt{\frac{-\frac{I}{2}\,b\left(2\,x+\frac{(-a\,b^2)^{1/3}+I\sqrt{3}\,(-a\,b^2)^{1/3}}{b}\right)}{(-a\,b^2)^{1/3}}}\,\big(I$$

$$(-a\,b^2)^{1/3}\sqrt{3}\,\_\alpha\,b-I\,(-a\,b^2)^{2/3}\sqrt{3}+2\,\_\alpha^2\,b^2-(-a\,b^2)^{1/3}\,\_\alpha\,b-(-a\,b^2)^{2/3}\big)$$

$$\mathrm{EllipticPi}\left(\frac{\sqrt{3}\sqrt{\frac{I\left(x+\frac{(-a\,b^2)^{1/3}}{2\,b}-\frac{I\sqrt{3}\,(-a\,b^2)^{1/3}}{2\,b}\right)\sqrt{3}\,b}{(-a\,b^2)^{1/3}}}}{3},\right.$$

$$-\frac{c^2\left(2\,I\sqrt{3}\,(-a\,b^2)^{1/3}\,\_\alpha^2\,b-I\sqrt{3}\,(-a\,b^2)^{2/3}\,\_\alpha+I\sqrt{3}\,a\,b-3\,(-a\,b^2)^{2/3}\,\_\alpha-3\,a\,b\right)}{2\,b\,d^2},\ \sqrt{\frac{I\sqrt{3}\,(-a\,b^2)^{1/3}}{b\left(-\frac{3\,(-a\,b^2)^{1/3}}{2\,b}+\frac{I\sqrt{3}\,(-a\,b^2)^{1/3}}{2\,b}\right)}}\Bigg)\Bigg)\Bigg)\Bigg)$$

$$-\frac{a\ln(x^3\,b\,c^2+c^2\,a-d^2)}{3\,c\,b^2}+\frac{x^3}{3\,b\,c}+\frac{d^2\ln(x^3\,b\,c^2+c^2\,a-d^2)}{3\,b^2\,c^3}$$

Problem 148: Result is not expressed in closed-form.

$$\int \frac{1}{x^4\left(a c+b c x^3+d \sqrt{b x^3+a}\right)}\, dx$$

Optimal(type 3, 138 leaves, 8 steps):

$$-\frac{b d\left(3 c^2 a-d^2\right) \operatorname{arctanh}\left(\frac{\sqrt{b x^3+a}}{\sqrt{a}}\right)}{3 a^{3/2}\left(c^2 a-d^2\right)^2}-\frac{b c^3 \ln(x)}{\left(c^2 a-d^2\right)^2}+\frac{2 b c^3 \ln\left(d+c \sqrt{b x^3+a}\right)}{3\left(c^2 a-d^2\right)^2}+\frac{-a c+d \sqrt{b x^3+a}}{3 a\left(c^2 a-d^2\right) x^3}$$

Result(type 7, 862 leaves):

$$-\frac{c}{3\left(c^2 a-d^2\right) x^3}-\frac{2 b c^3 \ln(x)}{\left(c^2 a-d^2\right)^2}+\frac{c b \ln(x) d^2}{a\left(c^2 a-d^2\right)^2}+\frac{a c^5 b \ln\left(x^3 b c^2+c^2 a-d^2\right)}{3\left(c^2 a-d^2\right)^2 d^2}+\frac{b c \ln(x)}{a\left(c^2 a-d^2\right)}-\frac{b c^3 \ln\left(x^3 b c^2+c^2 a-d^2\right)}{3\left(c^2 a-d^2\right) d^2}+\frac{d \sqrt{b x^3+a}}{3 a\left(c^2 a-d^2\right) x^3}$$

$$+\frac{d b \operatorname{arctanh}\left(\frac{\sqrt{b x^3+a}}{\sqrt{a}}\right)}{3 a^{3/2}\left(c^2 a-d^2\right)}-\frac{2 b c^4 \sqrt{b x^3+a}}{3\left(c^2 a-d^2\right)^2 d}-\frac{1}{3 b\left(c^2 a-d^2\right)^2 d}\Bigg(I c^4 \sqrt{2}\Bigg(\sum_{\_\alpha=RootOf\left(b c^2 \_Z^3+c^2 a-d^2\right)} \frac{1}{\sqrt{b x^3+a}}\Bigg((-a b^2)^{1/}$$

$$3\sqrt{\frac{\frac{I}{2} b\left(2 x+\frac{-I\sqrt{3}\,(-a b^2)^{1/3}+(-a b^2)^{1/3}}{b}\right)}{(-a b^2)^{1/3}}}\sqrt{\frac{b\left(x-\frac{(-a b^2)^{1/3}}{b}\right)}{-3(-a b^2)^{1/3}+I\sqrt{3}\,(-a b^2)^{1/3}}}\sqrt{\frac{-\frac{I}{2} b\left(2 x+\frac{(-a b^2)^{1/3}+I\sqrt{3}\,(-a b^2)^{1/3}}{b}\right)}{(-a b^2)^{1/3}}}\;\Big(I$$

$$(-a b^2)^{1/3}\sqrt{3}\,\_\alpha b-I(-a b^2)^{2/3}\sqrt{3}+2\,\_\alpha^2 b^2-(-a b^2)^{1/3}\,\_\alpha b-(-a b^2)^{2/3}\Big)$$

$$\operatorname{EllipticPi}\left(\frac{\sqrt{3}\sqrt{\frac{I\left(x+\frac{(-a b^2)^{1/3}}{2 b}-\frac{I\sqrt{3}\,(-a b^2)^{1/3}}{2 b}\right)\sqrt{3}\, b}{(-a b^2)^{1/3}}}}{3},\right.$$

$$-\frac{c^2\left(2 I\sqrt{3}\left(-a b^2\right)^{1/3}\_\alpha^2 b-I\sqrt{3}\left(-a b^2\right)^{2/3}\_\alpha+I\sqrt{3}\,a b-3\left(-a b^2\right)^{2/3}\_\alpha-3 a b\right)}{2 b d^2},\sqrt{\frac{I\sqrt{3}\left(-a b^2\right)^{1/3}}{b\left(-\frac{3\left(-a b^2\right)^{1/3}}{2 b}+\frac{I\sqrt{3}\left(-a b^2\right)^{1/3}}{2 b}\right)}}\Bigg)\Bigg)\Bigg)\Bigg)$$

$$+\frac{2 b\sqrt{b x^3+a}}{3 a^2 d}+\frac{4 d b\sqrt{b x^3+a}\;c^2}{3 a\left(c^2 a-d^2\right)^2}-\frac{2 b\sqrt{b x^3+a}\;d^3}{3 a^2\left(c^2 a-d^2\right)^2}-\frac{4 d b\operatorname{arctanh}\left(\frac{\sqrt{b x^3+a}}{\sqrt{a}}\right)c^2}{3\sqrt{a}\left(c^2 a-d^2\right)^2}+\frac{2 b\operatorname{arctanh}\left(\frac{\sqrt{b x^3+a}}{\sqrt{a}}\right)d^3}{3 a^{3/2}\left(c^2 a-d^2\right)^2}$$

Problem 149: Result is not expressed in closed-form.

$$\int\frac{x^3}{a c+b c x^3+d\sqrt{b x^3+a}}\,dx$$

Optimal(type 6, 251 leaves, 10 steps):

$$\frac{x}{b c}-\frac{\left(c^2 a-d^2\right)^{1/3}\ln\left(\left(c^2 a-d^2\right)^{1/3}+b^{1/3}c^{2/3}x\right)}{3 b^{4/3}c^{5/3}}+\frac{\left(c^2 a-d^2\right)^{1/3}\ln\left(\left(c^2 a-d^2\right)^{2/3}-b^{1/3}c^{2/3}\left(c^2 a-d^2\right)^{1/3}x+b^{2/3}c^{4/3}x^2\right)}{6 b^{4/3}c^{5/3}}$$

$$+\frac{\left(c^2 a-d^2\right)^{1/3}\arctan\left(\frac{\left(1-\frac{2 b^{1/3}c^{2/3}x}{\left(c^2 a-d^2\right)^{1/3}}\right)\sqrt{3}}{3}\right)\sqrt{3}}{3 b^{4/3}c^{5/3}}-\frac{d x^4\,AppellF1\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{b x^3}{a},-\frac{b c^2 x^3}{c^2 a-d^2}\right)\sqrt{1+\frac{b x^3}{a}}}{4\left(c^2 a-d^2\right)\sqrt{b x^3+a}}$$

Result(type 7, 1543 leaves):

$$\frac{1}{3 b^2 c^2\sqrt{b x^3+a}}\Bigg(2 I d\sqrt{3}\left(-a b^2\right)^{1/}$$

$${}^{3}\sqrt{\frac{I\left(x+\frac{\left(-a b^2\right)^{1/3}}{2 b}-\frac{I\sqrt{3}\left(-a b^2\right)^{1/3}}{2 b}\right)\sqrt{3}\,b}{\left(-a b^2\right)^{1/3}}}\sqrt{\frac{x-\frac{\left(-a b^2\right)^{1/3}}{b}}{-\frac{3\left(-a b^2\right)^{1/3}}{2 b}+\frac{I\sqrt{3}\left(-a b^2\right)^{1/3}}{2 b}}}$$

$$\sqrt{\frac{-\mathrm{I}\left(x+\frac{\left(-a\,b^{2}\right)^{1/3}}{2\,b}+\frac{\mathrm{I}\sqrt{3}\,\left(-a\,b^{2}\right)^{1/3}}{2\,b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{1/3}}}\;\mathrm{EllipticF}\Bigg(\frac{\sqrt{3}\,\sqrt{\frac{\mathrm{I}\left(x+\frac{\left(-a\,b^{2}\right)^{1/3}}{2\,b}-\frac{\mathrm{I}\sqrt{3}\,\left(-a\,b^{2}\right)^{1/3}}{2\,b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{1/3}}}}{3},$$

$$\sqrt{\frac{\mathrm{I}\sqrt{3}\,\left(-a\,b^{2}\right)^{1/3}}{b\left(-\frac{3\,\left(-a\,b^{2}\right)^{1/3}}{2\,b}+\frac{\mathrm{I}\sqrt{3}\,\left(-a\,b^{2}\right)^{1/3}}{2\,b}\right)}}\Bigg)\Bigg)+\frac{1}{3\,d\,b^{4}}\Bigg(\mathrm{I}\sqrt{2}\,\Bigg(\sum_{\_\alpha=RootOf\left(\_Z^{3}\,b\,c^{2}+c^{2}\,a-d^{2}\right)}\frac{1}{\_\alpha^{2}\,\sqrt{b\,x^{3}+a}}\Bigg(\left(-a\,b^{2}\right)^{1/}$$

$$3\sqrt{\frac{\frac{\mathrm{I}}{2}\,b\left(2\,x+\frac{-\mathrm{I}\sqrt{3}\,\left(-a\,b^{2}\right)^{1/3}+\left(-a\,b^{2}\right)^{1/3}}{b}\right)}{\left(-a\,b^{2}\right)^{1/3}}}\;\sqrt{\frac{b\left(x-\frac{\left(-a\,b^{2}\right)^{1/3}}{b}\right)}{-3\,\left(-a\,b^{2}\right)^{1/3}+\mathrm{I}\sqrt{3}\,\left(-a\,b^{2}\right)^{1/3}}}\;\sqrt{\frac{-\frac{\mathrm{I}}{2}\,b\left(2\,x+\frac{\left(-a\,b^{2}\right)^{1/3}+\mathrm{I}\sqrt{3}\,\left(-a\,b^{2}\right)^{1/3}}{b}\right)}{\left(-a\,b^{2}\right)^{1/3}}}\;\big(\mathrm{I}$$

$$\left(-a\,b^{2}\right)^{1/3}\sqrt{3}\,\_\alpha\,b-\mathrm{I}\,\left(-a\,b^{2}\right)^{2/3}\sqrt{3}+2\,\_\alpha^{2}\,b^{2}-\left(-a\,b^{2}\right)^{1/3}\,\_\alpha\,b-\left(-a\,b^{2}\right)^{2/3}\big)$$

$$\mathrm{EllipticPi}\Bigg(\frac{\sqrt{3}\,\sqrt{\frac{\mathrm{I}\left(x+\frac{\left(-a\,b^{2}\right)^{1/3}}{2\,b}-\frac{\mathrm{I}\sqrt{3}\,\left(-a\,b^{2}\right)^{1/3}}{2\,b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{1/3}}}}{3},$$

$$-\frac{c^{2}\left(2\,\mathrm{I}\sqrt{3}\,\left(-a\,b^{2}\right)^{1/3}\,\_\alpha^{2}\,b-\mathrm{I}\sqrt{3}\,\left(-a\,b^{2}\right)^{2/3}\,\_\alpha+\mathrm{I}\sqrt{3}\,a\,b-3\,\left(-a\,b^{2}\right)^{2/3}\,\_\alpha-3\,a\,b\right)}{2\,b\,d^{2}},\;\sqrt{\frac{\mathrm{I}\sqrt{3}\,\left(-a\,b^{2}\right)^{1/3}}{b\left(-\frac{3\,\left(-a\,b^{2}\right)^{1/3}}{2\,b}+\frac{\mathrm{I}\sqrt{3}\,\left(-a\,b^{2}\right)^{1/3}}{2\,b}\right)}}\Bigg)\Bigg)\Bigg)\Bigg)$$

$$a \Bigg) - \frac{1}{3 b^4 c^2} \Bigg(\mathrm{I}\, d \sqrt{2} \Bigg(\sum_{\_\alpha = RootOf(\_Z^3 b c^2 + c^2 a - d^2)} \frac{1}{\_\alpha^2 \sqrt{b x^3 + a}} \Bigg((-a b^2)^{1/}$$

$$3 \sqrt{\frac{\frac{\mathrm{I}}{2} b \left(2x + \frac{-\mathrm{I}\sqrt{3}\,(-a b^2)^{1/3} + (-a b^2)^{1/3}}{b}\right)}{(-a b^2)^{1/3}}} \sqrt{\frac{b \left(x - \frac{(-a b^2)^{1/3}}{b}\right)}{-3\,(-a b^2)^{1/3} + \mathrm{I}\sqrt{3}\,(-a b^2)^{1/3}}} \sqrt{\frac{-\frac{\mathrm{I}}{2} b \left(2x + \frac{(-a b^2)^{1/3} + \mathrm{I}\sqrt{3}\,(-a b^2)^{1/3}}{b}\right)}{(-a b^2)^{1/3}}} \,(\mathrm{I}$$

$$(-a b^2)^{1/3} \sqrt{3}\,\_\alpha b - \mathrm{I}\,(-a b^2)^{2/3} \sqrt{3} + 2\,\_\alpha^2 b^2 - (-a b^2)^{1/3}\,\_\alpha b - (-a b^2)^{2/3})$$

$$\mathrm{EllipticPi}\left(\frac{\sqrt{3} \sqrt{\frac{\mathrm{I}\left(x + \frac{(-a b^2)^{1/3}}{2b} - \frac{\mathrm{I}\sqrt{3}\,(-a b^2)^{1/3}}{2b}\right)\sqrt{3}\, b}{(-a b^2)^{1/3}}}}{3}, \right.$$

$$\left. -\frac{c^2 \left(2\mathrm{I}\sqrt{3}\,(-a b^2)^{1/3}\,\_\alpha^2 b - \mathrm{I}\sqrt{3}\,(-a b^2)^{2/3}\,\_\alpha + \mathrm{I}\sqrt{3}\, a b - 3\,(-a b^2)^{2/3}\,\_\alpha - 3 a b\right)}{2 b d^2}, \sqrt{\frac{\mathrm{I}\sqrt{3}\,(-a b^2)^{1/3}}{b\left(-\frac{3\,(-a b^2)^{1/3}}{2b} + \frac{\mathrm{I}\sqrt{3}\,(-a b^2)^{1/3}}{2b}\right)}} \right)\Bigg)\Bigg)\Bigg)$$

$$-\frac{a \ln\left(x + \left(\frac{c^2 a - d^2}{b c^2}\right)^{1/3}\right)}{3 c b^2 \left(\frac{c^2 a - d^2}{b c^2}\right)^{2/3}} + \frac{a \ln\left(x^2 - x\left(\frac{c^2 a - d^2}{b c^2}\right)^{1/3} + \left(\frac{c^2 a - d^2}{b c^2}\right)^{2/3}\right)}{6 c b^2 \left(\frac{c^2 a - d^2}{b c^2}\right)^{2/3}} - \frac{a \sqrt{3} \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c^2 a - d^2}{b c^2}\right)^{1/3}} - 1\right)}{3}\right)}{3 c b^2 \left(\frac{c^2 a - d^2}{b c^2}\right)^{2/3}} + \frac{x}{b c}$$

$$+\frac{d^2 \ln\left(x+\left(\frac{c^2 a-d^2}{b c^2}\right)^{1/3}\right)}{3 b^2 c^3 \left(\frac{c^2 a-d^2}{b c^2}\right)^{2/3}}-\frac{d^2 \ln\left(x^2-x\left(\frac{c^2 a-d^2}{b c^2}\right)^{1/3}+\left(\frac{c^2 a-d^2}{b c^2}\right)^{2/3}\right)}{6 b^2 c^3 \left(\frac{c^2 a-d^2}{b c^2}\right)^{2/3}}+\frac{d^2 \sqrt{3}\, \arctan\left(\frac{\sqrt{3}\left(\frac{2 x}{\left(\frac{c^2 a-d^2}{b c^2}\right)^{1/3}}-1\right)}{3}\right)}{3 b^2 c^3 \left(\frac{c^2 a-d^2}{b c^2}\right)^{2/3}}$$

Problem 150: Result is not expressed in closed-form.

$$\int \frac{x}{a c+b c x^3+d \sqrt{b x^3+a}}\, dx$$

Optimal(type 6, 243 leaves, 9 steps):

$$-\frac{\ln\left(\left(c^2 a-d^2\right)^{1/3}+b^{1/3} c^{2/3} x\right)}{3 b^{2/3} c^{1/3} \left(c^2 a-d^2\right)^{1/3}}+\frac{\ln\left(\left(c^2 a-d^2\right)^{2/3}-b^{1/3} c^{2/3} \left(c^2 a-d^2\right)^{1/3} x+b^{2/3} c^{4/3} x^2\right)}{6 b^{2/3} c^{1/3} \left(c^2 a-d^2\right)^{1/3}}-\frac{\arctan\left(\frac{\left(1-\frac{2 b^{1/3} c^{2/3} x}{\left(c^2 a-d^2\right)^{1/3}}\right)\sqrt{3}}{3}\right)\sqrt{3}}{3 b^{2/3} c^{1/3} \left(c^2 a-d^2\right)^{1/3}}$$
$$-\frac{d x^2 \mathit{AppellF1}\left(\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{c^2 a-d^2}\right)\sqrt{1+\frac{b x^3}{a}}}{2\left(c^2 a-d^2\right)\sqrt{b x^3+a}}$$

Result(type 7, 618 leaves):

$$-\frac{1}{3 d b^3}\left(I\sqrt{2}\left(\sum_{\_\alpha = RootOf(\_Z^3 b c^2+c^2 a-d^2)} \frac{1}{\_\alpha \sqrt{b x^3+a}}\left(\left(-a b^2\right)^{1/}\right.\right.\right.$$

$$\sqrt[3]{\frac{\frac{I}{2} b\left(2 x+\frac{-I\sqrt{3}\left(-a b^2\right)^{1/3}+\left(-a b^2\right)^{1/3}}{b}\right)}{\left(-a b^2\right)^{1/3}}}\sqrt{\frac{b\left(x-\frac{\left(-a b^2\right)^{1/3}}{b}\right)}{-3\left(-a b^2\right)^{1/3}+I\sqrt{3}\left(-a b^2\right)^{1/3}}}\sqrt{\frac{-\frac{I}{2} b\left(2 x+\frac{\left(-a b^2\right)^{1/3}+I\sqrt{3}\left(-a b^2\right)^{1/3}}{b}\right)}{\left(-a b^2\right)^{1/3}}}\,\left(I\right.$$

$$\left.\left(-a b^2\right)^{1/3}\sqrt{3}\,\_\alpha b-I\left(-a b^2\right)^{2/3}\sqrt{3}+2\,\_\alpha^2 b^2-\left(-a b^2\right)^{1/3}\_\alpha b-\left(-a b^2\right)^{2/3}\right)$$

$$\mathrm{EllipticPi}\left(\frac{\sqrt{3}\sqrt{\frac{\mathrm{I}\left(x+\frac{(-a b^2)^{1/3}}{2 b}-\frac{\mathrm{I}\sqrt{3}\,(-a b^2)^{1/3}}{2 b}\right)\sqrt{3}\, b}{(-a b^2)^{1/3}}}}{3},\right.$$

$$\left.-\frac{c^2\left(2\,\mathrm{I}\sqrt{3}\,(-a b^2)^{1/3}\,\_\alpha^2 b-\mathrm{I}\sqrt{3}\,(-a b^2)^{2/3}\,\_\alpha+\mathrm{I}\sqrt{3}\, a b-3\,(-a b^2)^{2/3}\,\_\alpha-3 a b\right)}{2 b d^2},\sqrt{\frac{\mathrm{I}\sqrt{3}\,(-a b^2)^{1/3}}{b\left(-\frac{3\,(-a b^2)^{1/3}}{2 b}+\frac{\mathrm{I}\sqrt{3}\,(-a b^2)^{1/3}}{2 b}\right)}}\right)\Bigg)\Bigg)\Bigg)\Bigg)$$

$$-\frac{\ln\left(x+\left(\frac{c^2 a-d^2}{b c^2}\right)^{1/3}\right)}{3 b c\left(\frac{c^2 a-d^2}{b c^2}\right)^{1/3}}+\frac{\ln\left(x^2-x\left(\frac{c^2 a-d^2}{b c^2}\right)^{1/3}+\left(\frac{c^2 a-d^2}{b c^2}\right)^{2/3}\right)}{6 b c\left(\frac{c^2 a-d^2}{b c^2}\right)^{1/3}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2 x}{\left(\frac{c^2 a-d^2}{b c^2}\right)^{1/3}}-1\right)}{3}\right)}{3 b c\left(\frac{c^2 a-d^2}{b c^2}\right)^{1/3}}$$

Problem 151: Result is not expressed in closed-form.

$$\int \frac{1}{x^2\left(a c+b c x^3+d\sqrt{b x^3+a}\right)}\, dx$$

Optimal(type 6, 261 leaves, 10 steps):

$$-\frac{c}{(c^2 a-d^2)\, x}+\frac{b^{1/3} c^{5/3}\ln\left((c^2 a-d^2)^{1/3}+b^{1/3} c^{2/3} x\right)}{3\,(c^2 a-d^2)^{4/3}}-\frac{b^{1/3} c^{5/3}\ln\left((c^2 a-d^2)^{2/3}-b^{1/3} c^{2/3}(c^2 a-d^2)^{1/3} x+b^{2/3} c^{4/3} x^2\right)}{6\,(c^2 a-d^2)^{4/3}}$$

$$+\frac{b^{1/3} c^{5/3}\arctan\left(\frac{\left(1-\frac{2 b^{1/3} c^{2/3} x}{(c^2 a-d^2)^{1/3}}\right)\sqrt{3}}{3}\right)\sqrt{3}}{3\,(c^2 a-d^2)^{4/3}}+\frac{d\, \mathit{AppellF1}\left(-\frac{1}{3},\frac{1}{2},1,\frac{2}{3},-\frac{b x^3}{a},-\frac{b c^2 x^3}{c^2 a-d^2}\right)\sqrt{1+\frac{b x^3}{a}}}{(c^2 a-d^2)\, x\sqrt{b x^3+a}}$$

Result(type ?, 3559 leaves): Display of huge result suppressed!

Problem 152: Result is not expressed in closed-form.

$$\int \frac{1}{x^3\left(a c+b c x^3+d\sqrt{b x^3+a}\right)}\, dx$$

Optimal(type 6, 262 leaves, 10 steps):

$$-\frac{c}{2\left(c^{2} a-d^{2}\right) x^{2}}-\frac{b^{2/3} c^{7/3} \ln\left(\left(c^{2} a-d^{2}\right)^{1/3}+b^{1/3} c^{2/3} x\right)}{3\left(c^{2} a-d^{2}\right)^{5/3}}+\frac{b^{2/3} c^{7/3} \ln\left(\left(c^{2} a-d^{2}\right)^{2/3}-b^{1/3} c^{2/3}\left(c^{2} a-d^{2}\right)^{1/3} x+b^{2/3} c^{4/3} x^{2}\right)}{6\left(c^{2} a-d^{2}\right)^{5/3}}$$

$$+\frac{b^{2/3} c^{7/3} \arctan\left(\frac{\left(1-\frac{2 b^{1/3} c^{2/3} x}{\left(c^{2} a-d^{2}\right)^{1/3}}\right) \sqrt{3}}{3}\right) \sqrt{3}}{3\left(c^{2} a-d^{2}\right)^{5/3}}+\frac{d\, AppellF1\left(-\frac{2}{3}, \frac{1}{2}, 1, \frac{1}{3}, -\frac{b x^{3}}{a}, -\frac{b c^{2} x^{3}}{c^{2} a-d^{2}}\right) \sqrt{1+\frac{b x^{3}}{a}}}{2\left(c^{2} a-d^{2}\right) x^{2} \sqrt{b x^{3}+a}}$$

Result(type 7, 1788 leaves):

$$\frac{c \ln\left(x+\left(\frac{c^{2} a-d^{2}}{b c^{2}}\right)^{1/3}\right)}{3 d^{2}\left(\frac{c^{2} a-d^{2}}{b c^{2}}\right)^{2/3}}-\frac{c \ln\left(x^{2}-x\left(\frac{c^{2} a-d^{2}}{b c^{2}}\right)^{1/3}+\left(\frac{c^{2} a-d^{2}}{b c^{2}}\right)^{2/3}\right)}{6 d^{2}\left(\frac{c^{2} a-d^{2}}{b c^{2}}\right)^{2/3}}+\frac{c \sqrt{3} \arctan\left(\frac{\sqrt{3}\left(\frac{2 x}{\left(\frac{c^{2} a-d^{2}}{b c^{2}}\right)^{1/3}}-1\right)}{3}\right)}{3 d^{2}\left(\frac{c^{2} a-d^{2}}{b c^{2}}\right)^{2/3}}-\frac{c}{2\left(c^{2} a-d^{2}\right) x^{2}}$$

$$-\frac{a c^{3} \ln\left(x+\left(\frac{c^{2} a-d^{2}}{b c^{2}}\right)^{1/3}\right)}{3 d^{2}\left(c^{2} a-d^{2}\right)\left(\frac{c^{2} a-d^{2}}{b c^{2}}\right)^{2/3}}+\frac{a c^{3} \ln\left(x^{2}-x\left(\frac{c^{2} a-d^{2}}{b c^{2}}\right)^{1/3}+\left(\frac{c^{2} a-d^{2}}{b c^{2}}\right)^{2/3}\right)}{6 d^{2}\left(c^{2} a-d^{2}\right)\left(\frac{c^{2} a-d^{2}}{b c^{2}}\right)^{2/3}}-\frac{a c^{3} \sqrt{3} \arctan\left(\frac{\sqrt{3}\left(\frac{2 x}{\left(\frac{c^{2} a-d^{2}}{b c^{2}}\right)^{1/3}}-1\right)}{3}\right)}{3 d^{2}\left(c^{2} a-d^{2}\right)\left(\frac{c^{2} a-d^{2}}{b c^{2}}\right)^{2/3}}$$

$$+\frac{d \sqrt{b x^{3}+a}}{2 a\left(c^{2} a-d^{2}\right) x^{2}}+\frac{1}{2 a\left(c^{2} a-d^{2}\right) \sqrt{b x^{3}+a}}\left(\mathrm{I}\, d \sqrt{3}\left(-a b^{2}\right)^{1/}\right.$$

$$3 \sqrt{\frac{\mathrm{I}\left(x+\frac{\left(-a b^{2}\right)^{1/3}}{2 b}-\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1/3}}{2 b}\right) \sqrt{3}\, b}{\left(-a b^{2}\right)^{1/3}}} \sqrt{\frac{x-\frac{\left(-a b^{2}\right)^{1/3}}{b}}{-\frac{3\left(-a b^{2}\right)^{1/3}}{2 b}+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1/3}}{2 b}}}$$

$$\sqrt{\frac{-\mathrm{I}\left(x+\frac{\left(-a b^{2}\right)^{1/3}}{2 b}+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1/3}}{2 b}\right) \sqrt{3}\, b}{\left(-a b^{2}\right)^{1/3}}}\ \mathrm{EllipticF}\left(\frac{\sqrt{3} \sqrt{\frac{\mathrm{I}\left(x+\frac{\left(-a b^{2}\right)^{1/3}}{2 b}-\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1/3}}{2 b}\right) \sqrt{3}\, b}{\left(-a b^{2}\right)^{1/3}}}}{3},\right.$$

$$\sqrt{\frac{I\sqrt{3}\,(-a\,b^2)^{1/3}}{b\left(-\frac{3\,(-a\,b^2)^{1/3}}{2\,b}+\frac{I\sqrt{3}\,(-a\,b^2)^{1/3}}{2\,b}\right)}}\Bigg)\Bigg)+\frac{1}{3\,a\,d\sqrt{b\,x^3+a}}\Bigg(2\,I\sqrt{3}\,(-a\,b^2)^{1/}$$

$$3\sqrt{\frac{I\left(x+\frac{(-a\,b^2)^{1/3}}{2\,b}-\frac{I\sqrt{3}\,(-a\,b^2)^{1/3}}{2\,b}\right)\sqrt{3}\,b}{(-a\,b^2)^{1/3}}}\sqrt{\frac{x-\frac{(-a\,b^2)^{1/3}}{b}}{-\frac{3\,(-a\,b^2)^{1/3}}{2\,b}+\frac{I\sqrt{3}\,(-a\,b^2)^{1/3}}{2\,b}}}$$

$$\sqrt{\frac{-I\left(x+\frac{(-a\,b^2)^{1/3}}{2\,b}+\frac{I\sqrt{3}\,(-a\,b^2)^{1/3}}{2\,b}\right)\sqrt{3}\,b}{(-a\,b^2)^{1/3}}}\ \mathrm{EllipticF}\left(\frac{\sqrt{3}\sqrt{\frac{I\left(x+\frac{(-a\,b^2)^{1/3}}{2\,b}-\frac{I\sqrt{3}\,(-a\,b^2)^{1/3}}{2\,b}\right)\sqrt{3}\,b}{(-a\,b^2)^{1/3}}}}{3},\right.$$

$$\sqrt{\frac{I\sqrt{3}\,(-a\,b^2)^{1/3}}{b\left(-\frac{3\,(-a\,b^2)^{1/3}}{2\,b}+\frac{I\sqrt{3}\,(-a\,b^2)^{1/3}}{2\,b}\right)}}\Bigg)\Bigg)-\frac{1}{3\,d\,(c^2\,a-d^2)\sqrt{b\,x^3+a}}\Bigg(2\,I\,c^2\sqrt{3}\,(-a\,b^2)^{1/}$$

$$3\sqrt{\frac{I\left(x+\frac{(-a\,b^2)^{1/3}}{2\,b}-\frac{I\sqrt{3}\,(-a\,b^2)^{1/3}}{2\,b}\right)\sqrt{3}\,b}{(-a\,b^2)^{1/3}}}\sqrt{\frac{x-\frac{(-a\,b^2)^{1/3}}{b}}{-\frac{3\,(-a\,b^2)^{1/3}}{2\,b}+\frac{I\sqrt{3}\,(-a\,b^2)^{1/3}}{2\,b}}}$$

$$\sqrt{\frac{-I\left(x+\frac{(-a\,b^2)^{1/3}}{2\,b}+\frac{I\sqrt{3}\,(-a\,b^2)^{1/3}}{2\,b}\right)\sqrt{3}\,b}{(-a\,b^2)^{1/3}}}\ \mathrm{EllipticF}\left(\frac{\sqrt{3}\sqrt{\frac{I\left(x+\frac{(-a\,b^2)^{1/3}}{2\,b}-\frac{I\sqrt{3}\,(-a\,b^2)^{1/3}}{2\,b}\right)\sqrt{3}\,b}{(-a\,b^2)^{1/3}}}}{3},\right.$$

$$\sqrt{\frac{\mathrm{I}\sqrt{3}\,(-a\,b^2)^{1/3}}{b\left(-\frac{3\,(-a\,b^2)^{1/3}}{2\,b}+\frac{\mathrm{I}\sqrt{3}\,(-a\,b^2)^{1/3}}{2\,b}\right)}}\Bigg)\Bigg)+\frac{1}{3\,b^2\,d\,(c^2\,a-d^2)}\Bigg(\mathrm{I}\,c^2\sqrt{2}\Bigg(\sum_{\_\alpha=RootOf(\_Z^3\,b\,c^2+c^2\,a-d^2)}\frac{1}{\_\alpha^2\sqrt{b\,x^3+a}}\Bigg(($$

$$-a\,b^2)^{1/}$$

$$3\sqrt{\frac{\frac{\mathrm{I}}{2}\,b\left(2\,x+\frac{-\mathrm{I}\sqrt{3}\,(-a\,b^2)^{1/3}+(-a\,b^2)^{1/3}}{b}\right)}{(-a\,b^2)^{1/3}}}\sqrt{\frac{b\left(x-\frac{(-a\,b^2)^{1/3}}{b}\right)}{-3\,(-a\,b^2)^{1/3}+\mathrm{I}\sqrt{3}\,(-a\,b^2)^{1/3}}}\sqrt{\frac{-\frac{\mathrm{I}}{2}\,b\left(2\,x+\frac{(-a\,b^2)^{1/3}+\mathrm{I}\sqrt{3}\,(-a\,b^2)^{1/3}}{b}\right)}{(-a\,b^2)^{1/3}}}\,(\mathrm{I}$$

$$(-a\,b^2)^{1/3}\sqrt{3}\,\_\alpha\,b-\mathrm{I}\,(-a\,b^2)^{2/3}\sqrt{3}+2\,\_\alpha^2\,b^2-(-a\,b^2)^{1/3}\,\_\alpha\,b-(-a\,b^2)^{2/3})$$

$$\mathrm{EllipticPi}\left(\frac{\sqrt{3}\sqrt{\frac{\mathrm{I}\left(x+\frac{(-a\,b^2)^{1/3}}{2\,b}-\frac{\mathrm{I}\sqrt{3}\,(-a\,b^2)^{1/3}}{2\,b}\right)\sqrt{3}\,b}{(-a\,b^2)^{1/3}}}}{3},\right.$$

$$\left.-\frac{c^2\left(2\,\mathrm{I}\sqrt{3}\,(-a\,b^2)^{1/3}\,\_\alpha^2\,b-\mathrm{I}\sqrt{3}\,(-a\,b^2)^{2/3}\,\_\alpha+\mathrm{I}\sqrt{3}\,a\,b-3\,(-a\,b^2)^{2/3}\,\_\alpha-3\,a\,b\right)}{2\,b\,d^2},\sqrt{\frac{\mathrm{I}\sqrt{3}\,(-a\,b^2)^{1/3}}{b\left(-\frac{3\,(-a\,b^2)^{1/3}}{2\,b}+\frac{\mathrm{I}\sqrt{3}\,(-a\,b^2)^{1/3}}{2\,b}\right)}}\right)\Bigg)\Bigg)\Bigg)\Bigg)$$

Problem 154: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^{1/4}+x^{1/3}}\,dx$$

Optimal(type 3, 49 leaves, 4 steps):

$$-12x^{1/12}+6x^{1/6}-4x^{1/4}+3x^{1/3}-\frac{12x^{5/12}}{5}-\frac{12x^{7/12}}{7}+\frac{3x^{2/3}}{2}+12\ln\left(1+x^{1/12}\right)+2\sqrt{x}$$

Result(type 3, 172 leaves):

$$3x^{1/3}+2\sqrt{x}-4x^{1/4}-2\ln\left(x^{1/4}-1\right)-\ln\left(x^{2/3}+x^{1/3}+1\right)-\frac{12x^{7/12}}{7}+6x^{1/6}-12x^{1/12}-\frac{12x^{5/12}}{5}+2\ln\left(x^{1/3}-1\right)+\ln\left(-1+\sqrt{x}\right)-\ln\left(1+\sqrt{x}\right)-2\ln\left(x^{1/6}+1\right)+2\ln\left(x^{1/6}-1\right)+\ln\left(x^{1/3}-x^{1/6}+1\right)+4\ln\left(1+x^{1/12}\right)-4\ln\left(x^{1/12}-1\right)-2\ln\left(x^{1/6}-x^{1/12}+1\right)+2\ln\left(x^{1/6}+x^{1/12}+1\right)+2\ln\left(x^{1/4}+1\right)-\ln\left(x^{1/3}+x^{1/6}+1\right)+\ln(-1+x)+\frac{3x^{2/3}}{2}$$

Problem 156: Unable to integrate problem.

$$\int \left(a+\frac{b}{x}\right)^m dx$$

Optimal(type 5, 42 leaves, 2 steps):

$$-\frac{b\left(a+\frac{b}{x}\right)^{1+m}\mathrm{hypergeom}\left([2,1+m],[2+m],1+\frac{b}{a\,x}\right)}{a^2\,(1+m)}$$

Result(type 8, 11 leaves):

$$\int \left(a+\frac{b}{x}\right)^m dx$$

Problem 157: Unable to integrate problem.

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(d\,x+c)^3}\,dx$$

Optimal(type 5, 110 leaves, 4 steps):

$$-\frac{d\left(a+\frac{b}{x}\right)^{1+m}}{2\,c\,(a\,c-b\,d)\left(d+\frac{c}{x}\right)^2}-\frac{b\,(2\,a\,c-b\,d\,(1+m))\left(a+\frac{b}{x}\right)^{1+m}\mathrm{hypergeom}\left([2,1+m],[2+m],\frac{c\left(a+\frac{b}{x}\right)}{a\,c-b\,d}\right)}{2\,c\,(a\,c-b\,d)^3\,(1+m)}$$

Result(type 8, 19 leaves):

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(d x+c)^3}\, dx$$

Problem 159: Result more than twice size of optimal antiderivative.

$$\int \frac{(d x+c)^{3/2}}{\sqrt{a+\frac{b}{x^2}}}\, dx$$

Optimal(type 4, 330 leaves, 8 steps):

$$\frac{2 (d x+c)^{3/2} \left(a x^2+b\right)}{5 a x \sqrt{a+\frac{b}{x^2}}}+\frac{2 c \left(a x^2+b\right) \sqrt{d x+c}}{5 a x \sqrt{a+\frac{b}{x^2}}}$$

$$+\frac{2 \left(c^2 a-3 d^2 b\right) \mathrm{EllipticE}\left(\frac{\sqrt{1-\frac{x \sqrt{-a}}{\sqrt{b}}}\, \sqrt{2}}{2}, \sqrt{-\frac{2 d \sqrt{-a}\, \sqrt{b}}{a c-d \sqrt{-a}\, \sqrt{b}}}\right) \sqrt{b}\, \sqrt{d x+c}\, \sqrt{1+\frac{a x^2}{b}}}{5 (-a)^{3/2} d x \sqrt{a+\frac{b}{x^2}}\, \sqrt{\frac{a (d x+c)}{a c-d \sqrt{-a}\, \sqrt{b}}}}$$

$$-\frac{2 c \left(c^2 a+d^2 b\right) \mathrm{EllipticF}\left(\frac{\sqrt{1-\frac{x \sqrt{-a}}{\sqrt{b}}}\, \sqrt{2}}{2}, \sqrt{-\frac{2 d \sqrt{-a}\, \sqrt{b}}{a c-d \sqrt{-a}\, \sqrt{b}}}\right) \sqrt{b}\, \sqrt{1+\frac{a x^2}{b}}\, \sqrt{\frac{a (d x+c)}{a c-d \sqrt{-a}\, \sqrt{b}}}}{5 (-a)^{3/2} d x \sqrt{a+\frac{b}{x^2}}\, \sqrt{d x+c}}$$

Result(type 4, 1144 leaves):

$$\frac{1}{5 \sqrt{d x+c}\, d^2 a^2 x \sqrt{\frac{a x^2+b}{x^2}}} \Bigg(2 \Bigg(\sqrt{-a b}\, \sqrt{-\frac{(d x+c) a}{\sqrt{-a b}\, d-a c}}\, \sqrt{\frac{\left(-a x+\sqrt{-a b}\right) d}{\sqrt{-a b}\, d+a c}}\, \sqrt{\frac{\left(a x+\sqrt{-a b}\right) d}{\sqrt{-a b}\, d-a c}}\, \mathrm{EllipticF}\left(\sqrt{-\frac{(d x+c) a}{\sqrt{-a b}\, d-a c}},\right.$$

$$\left.\sqrt{-\frac{\sqrt{-a b}\, d-a c}{\sqrt{-a b}\, d+a c}}\right) a c^3 d+\sqrt{-a b}\, \sqrt{-\frac{(d x+c) a}{\sqrt{-a b}\, d-a c}}\, \sqrt{\frac{\left(-a x+\sqrt{-a b}\right) d}{\sqrt{-a b}\, d+a c}}\, \sqrt{\frac{\left(a x+\sqrt{-a b}\right) d}{\sqrt{-a b}\, d-a c}}\, \mathrm{EllipticF}\left(\sqrt{-\frac{(d x+c) a}{\sqrt{-a b}\, d-a c}},\right.$$

$$\left.\sqrt{-\frac{\sqrt{-a b}\, d-a c}{\sqrt{-a b}\, d+a c}}\right) b c d^3-3 \sqrt{-\frac{(d x+c) a}{\sqrt{-a b}\, d-a c}}\, \sqrt{\frac{\left(-a x+\sqrt{-a b}\right) d}{\sqrt{-a b}\, d+a c}}\, \sqrt{\frac{\left(a x+\sqrt{-a b}\right) d}{\sqrt{-a b}\, d-a c}}\, \mathrm{EllipticF}\left(\sqrt{-\frac{(d x+c) a}{\sqrt{-a b}\, d-a c}},\right.$$

$$\sqrt{-\frac{\sqrt{-ab}\,d-ac}{\sqrt{-ab}\,d+ac}}\right) a b c^2 d^2 - 3 b^2 \sqrt{-\frac{(dx+c)\,a}{\sqrt{-ab}\,d-ac}} \sqrt{\frac{\left(-ax+\sqrt{-ab}\right)d}{\sqrt{-ab}\,d+ac}} \sqrt{\frac{\left(ax+\sqrt{-ab}\right)d}{\sqrt{-ab}\,d-ac}}\ \mathrm{EllipticF}\left(\sqrt{-\frac{(dx+c)\,a}{\sqrt{-ab}\,d-ac}},\right.$$

$$\left.\sqrt{-\frac{\sqrt{-ab}\,d-ac}{\sqrt{-ab}\,d+ac}}\right) d^4 - \sqrt{-\frac{(dx+c)\,a}{\sqrt{-ab}\,d-ac}} \sqrt{\frac{\left(-ax+\sqrt{-ab}\right)d}{\sqrt{-ab}\,d+ac}} \sqrt{\frac{\left(ax+\sqrt{-ab}\right)d}{\sqrt{-ab}\,d-ac}}\ \mathrm{EllipticE}\left(\sqrt{-\frac{(dx+c)\,a}{\sqrt{-ab}\,d-ac}}, \sqrt{-\frac{\sqrt{-ab}\,d-ac}{\sqrt{-ab}\,d+ac}}\right) a^2 c^4$$

$$+ 2 \sqrt{-\frac{(dx+c)\,a}{\sqrt{-ab}\,d-ac}} \sqrt{\frac{\left(-ax+\sqrt{-ab}\right)d}{\sqrt{-ab}\,d+ac}} \sqrt{\frac{\left(ax+\sqrt{-ab}\right)d}{\sqrt{-ab}\,d-ac}}\ \mathrm{EllipticE}\left(\sqrt{-\frac{(dx+c)\,a}{\sqrt{-ab}\,d-ac}}, \sqrt{-\frac{\sqrt{-ab}\,d-ac}{\sqrt{-ab}\,d+ac}}\right) a b c^2 d^2$$

$$+ 3 b^2 \sqrt{-\frac{(dx+c)\,a}{\sqrt{-ab}\,d-ac}} \sqrt{\frac{\left(-ax+\sqrt{-ab}\right)d}{\sqrt{-ab}\,d+ac}} \sqrt{\frac{\left(ax+\sqrt{-ab}\right)d}{\sqrt{-ab}\,d-ac}}\ \mathrm{EllipticE}\left(\sqrt{-\frac{(dx+c)\,a}{\sqrt{-ab}\,d-ac}}, \sqrt{-\frac{\sqrt{-ab}\,d-ac}{\sqrt{-ab}\,d+ac}}\right) d^4 + x^4 a^2 d^4 + 3 x^3 a^2 c d^3$$

$$\left.\left. + 2 x^2 a^2 c^2 d^2 + x^2 a b d^4 + 3 x a b c d^3 + 2 a b c^2 d^2 \right)\right)$$

Problem 174: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^2 \sqrt{a + b\sqrt{dx+c}}}\, dx$$

Optimal(type 3, 125 leaves, 7 steps):

$$-\frac{b\,d\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{a-b\sqrt{c}}}\right)}{2\sqrt{c}\left(a-b\sqrt{c}\right)^{3/2}} + \frac{b\,d\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{a+b\sqrt{c}}}\right)}{2\sqrt{c}\left(a+b\sqrt{c}\right)^{3/2}} - \frac{\left(a-b\sqrt{dx+c}\right)\sqrt{a+b\sqrt{dx+c}}}{\left(-b^2 c + a^2\right) x}$$

Result(type 3, 264 leaves):

$$-\frac{2 d \sqrt{b^2 c}\,\sqrt{a+b\sqrt{dx+c}}}{c\left(-4\sqrt{b^2 c} - 4a\right)\left(-b\sqrt{dx+c} + \sqrt{b^2 c}\right)} + \frac{2 d \sqrt{b^2 c}\ \arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{-\sqrt{b^2 c} - a}}\right)}{c\left(-4\sqrt{b^2 c} - 4a\right)\sqrt{-\sqrt{b^2 c} - a}} - \frac{2 d \sqrt{b^2 c}\,\sqrt{a+b\sqrt{dx+c}}}{c\left(4\sqrt{b^2 c} - 4a\right)\left(b\sqrt{dx+c} + \sqrt{b^2 c}\right)}$$

$$-\frac{2 d \sqrt{b^2 c}\ \arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{\sqrt{b^2 c} - a}}\right)}{c\left(4\sqrt{b^2 c} - 4a\right)\sqrt{\sqrt{b^2 c} - a}}$$

Problem 175: Unable to integrate problem.

$$\int\left(a+b\sqrt{dx+c}\right)^{p}dx$$

Optimal(type 3, 58 leaves, 4 steps):

$$-\frac{2a\left(a+b\sqrt{dx+c}\right)^{1+p}}{b^{2}d\left(1+p\right)}+\frac{2\left(a+b\sqrt{dx+c}\right)^{2+p}}{b^{2}d\left(2+p\right)}$$

Result(type 8, 15 leaves):

$$\int\left(a+b\sqrt{dx+c}\right)^{p}dx$$

Problem 176: Unable to integrate problem.

$$\int\frac{\left(a+b\sqrt{dx+c}\right)^{p}}{x}dx$$

Optimal(type 5, 127 leaves, 6 steps):

$$-\frac{\mathrm{hypergeom}\left(\left[1,1+p\right],\left[2+p\right],\frac{a+b\sqrt{dx+c}}{a-b\sqrt{c}}\right)\left(a+b\sqrt{dx+c}\right)^{1+p}}{\left(1+p\right)\left(a-b\sqrt{c}\right)}-\frac{\mathrm{hypergeom}\left(\left[1,1+p\right],\left[2+p\right],\frac{a+b\sqrt{dx+c}}{a+b\sqrt{c}}\right)\left(a+b\sqrt{dx+c}\right)^{1+p}}{\left(1+p\right)\left(a+b\sqrt{c}\right)}$$

Result(type 8, 19 leaves):

$$\int\frac{\left(a+b\sqrt{dx+c}\right)^{p}}{x}dx$$

Problem 191: Result more than twice size of optimal antiderivative.

$$\int\frac{1}{x-\sqrt{1-x}}dx$$

Optimal(type 3, 49 leaves, 4 steps):

$$\frac{\ln\left(1-\sqrt{5}+2\sqrt{1-x}\right)\left(5-\sqrt{5}\right)}{5}+\frac{\ln\left(1+\sqrt{5}+2\sqrt{1-x}\right)\left(5+\sqrt{5}\right)}{5}$$

Result(type 3, 100 leaves):

$$\frac{\ln\left(x^{2}+x-1\right)}{2}+\frac{\sqrt{5}\,\mathrm{arctanh}\left(\frac{\left(1+2x\right)\sqrt{5}}{5}\right)}{5}+\frac{\ln\left(-x+\sqrt{1-x}\right)}{2}+\frac{\sqrt{5}\,\mathrm{arctanh}\left(\frac{\left(2\sqrt{1-x}+1\right)\sqrt{5}}{5}\right)}{5}-\frac{\ln\left(-x-\sqrt{1-x}\right)}{2}$$
$$+\frac{\sqrt{5}\,\mathrm{arctanh}\left(\frac{\left(2\sqrt{1-x}-1\right)\sqrt{5}}{5}\right)}{5}$$

Problem 195: Result more than twice size of optimal antiderivative.

$$\int \frac{x^3-1}{(x^2+1)\sqrt{x}}\,dx$$

Optimal(type 3, 36 leaves, 8 steps):

$$\frac{2x^{3/2}}{3}-\arctan\left(\sqrt{2}\sqrt{x}-1\right)\sqrt{2}-\arctan\left(1+\sqrt{2}\sqrt{x}\right)\sqrt{2}$$

Result(type 3, 96 leaves):

$$\frac{2x^{3/2}}{3}-\arctan\left(1+\sqrt{2}\sqrt{x}\right)\sqrt{2}-\arctan\left(\sqrt{2}\sqrt{x}-1\right)\sqrt{2}-\frac{\sqrt{2}\ln\left(\frac{x+\sqrt{2}\sqrt{x}+1}{x-\sqrt{2}\sqrt{x}+1}\right)}{4}-\frac{\sqrt{2}\ln\left(\frac{x-\sqrt{2}\sqrt{x}+1}{x+\sqrt{2}\sqrt{x}+1}\right)}{4}$$

Problem 201: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{\frac{x}{1+x}}}{x}\,dx$$

Optimal(type 3, 6 leaves, 3 steps):

$$2\,\mathrm{arcsinh}\left(\sqrt{x}\right)$$

Result(type 3, 31 leaves):

$$\frac{\sqrt{\frac{x}{1+x}}\,(1+x)\ln\left(\frac{1}{2}+x+\sqrt{x^2+x}\right)}{\sqrt{(1+x)\,x}}$$

Problem 202: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{-\frac{x}{1+x}}}{x}\,dx$$

Optimal(type 3, 13 leaves, 2 steps):

$$2\arctan\left(\sqrt{-\frac{x}{1+x}}\right)$$

Result(type 3, 32 leaves):

$$\frac{\sqrt{-\frac{x}{1+x}}\,(1+x)\ln\left(\frac{1}{2}+x+\sqrt{x^2+x}\right)}{\sqrt{(1+x)\,x}}$$

Problem 203: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{\frac{bx+a}{dx+c}}}{bx+a}\, dx$$

Optimal(type 3, 31 leaves, 3 steps):

$$\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{d}\,\sqrt{\frac{bx+a}{dx+c}}}{\sqrt{b}}\right)}{\sqrt{b}\,\sqrt{d}}$$

Result(type 3, 79 leaves):

$$\frac{\ln\left(\frac{2bdx+2\sqrt{(bx+a)(dx+c)}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)(dx+c)\sqrt{\frac{bx+a}{dx+c}}}{\sqrt{(bx+a)(dx+c)}\,\sqrt{bd}}$$

Problem 206: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\left(x+\sqrt{-x^2-2x+3}\right)^3}\, dx$$

Optimal(type 3, 248 leaves, 6 steps):

$$\frac{12\,\mathrm{arctanh}\left(\frac{\left(3-x-x\sqrt{3}-\sqrt{3}\,\sqrt{-x^2-2x+3}\right)\sqrt{7}}{7x}\right)\sqrt{7}}{343}$$
$$-\frac{4\left(9-5\sqrt{3}+\frac{\left(21+5\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}\right)}{21\left(2-\sqrt{3}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}+\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^2}{x^2}\right)^2}$$
$$+\frac{2\left(18-43\sqrt{3}-\frac{\left(18+49\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}\right)}{147\left(2-\sqrt{3}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}+\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^2}{x^2}\right)}$$

Result(type ?, 5999 leaves): Display of huge result suppressed!

Problem 209: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\left(x+\sqrt{-x^2-4x-3}\right)^2}\, dx$$

Optimal(type 3, 73 leaves, 5 steps):

$$\frac{\arctan\left(\frac{\left(1-\frac{3\sqrt{-1-x}}{\sqrt{3+x}}\right)\sqrt{2}}{2}\right)\sqrt{2}}{2}+\frac{1-\frac{\sqrt{-1-x}}{\sqrt{3+x}}}{1-\frac{3(1+x)}{3+x}-\frac{2\sqrt{-1-x}}{\sqrt{3+x}}}$$

Result(type ?, 2406 leaves): Display of huge result suppressed!

Problem 211: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\left(-x^4+4x^3-8x^2+8x\right)^{5/2}}\, dx$$

Optimal(type 4, 93 leaves, 7 steps):

$$\frac{\left(5+(-1+x)^2\right)(-1+x)}{72\left(3-2(-1+x)^2-(-1+x)^4\right)^{3/2}}-\frac{7\,\mathrm{EllipticE}\left(-1+x, \frac{I}{3}\sqrt{3}\right)\sqrt{3}}{432}+\frac{11\,\mathrm{EllipticF}\left(-1+x, \frac{I}{3}\sqrt{3}\right)\sqrt{3}}{432}+\frac{\left(26+7(-1+x)^2\right)(-1+x)}{432\sqrt{3-2(-1+x)^2-(-1+x)^4}}$$

Result(type 4, 1038 leaves):

$$-\frac{\sqrt{-x^4+4x^3-8x^2+8x}}{768x^2}-\frac{-x^3+4x^2-8x+8}{96\sqrt{x\left(-x^3+4x^2-8x+8\right)}}+\frac{\left(\frac{1}{36}+\frac{1}{288}x^2-\frac{1}{96}x\right)\sqrt{-x^4+4x^3-8x^2+8x}}{\left(x^3-4x^2+8x-8\right)^2}+\frac{2x\left(\frac{53}{3456}+\frac{5}{1728}x^2-\frac{19}{4608}x\right)}{\sqrt{-x\left(x^3-4x^2+8x-8\right)}}$$

$$+\frac{1}{216\left(-1-I\sqrt{3}\right)\sqrt{-x(-2+x)\left(x-I\sqrt{3}-1\right)\left(x-1+I\sqrt{3}\right)}}\left(5\left(I\sqrt{3}-1\right)\sqrt{\frac{\left(-1-I\sqrt{3}\right)x}{\left(1-I\sqrt{3}\right)(-2+x)}}\,(-2\right.$$

$$\left.+x)^2\sqrt{\frac{x-I\sqrt{3}-1}{\left(I\sqrt{3}+1\right)(-2+x)}}\sqrt{\frac{x-1+I\sqrt{3}}{\left(1-I\sqrt{3}\right)(-2+x)}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(-1-I\sqrt{3}\right)x}{\left(1-I\sqrt{3}\right)(-2+x)}}, \sqrt{\frac{\left(1-I\sqrt{3}\right)\left(I\sqrt{3}-1\right)}{\left(-1-I\sqrt{3}\right)\left(I\sqrt{3}+1\right)}}\right)\right)$$

$$+\frac{1}{108\left(-1-I\sqrt{3}\right)\sqrt{-x(-2+x)\left(x-I\sqrt{3}-1\right)\left(x-1+I\sqrt{3}\right)}}\left(7\left(I\sqrt{3}-1\right)\sqrt{\frac{\left(-1-I\sqrt{3}\right)x}{\left(1-I\sqrt{3}\right)(-2+x)}}\,(-2\right.$$

$$+x)^2\sqrt{\frac{x-I\sqrt{3}-1}{\left(I\sqrt{3}+1\right)(-2+x)}}\sqrt{\frac{x-1+I\sqrt{3}}{\left(1-I\sqrt{3}\right)(-2+x)}}\left(2\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(-1-I\sqrt{3}\right)x}{\left(1-I\sqrt{3}\right)(-2+x)}}, \sqrt{\frac{\left(1-I\sqrt{3}\right)\left(I\sqrt{3}-1\right)}{\left(-1-I\sqrt{3}\right)\left(I\sqrt{3}+1\right)}}\right)\right.$$

$$\left.\left.-2\,\mathrm{EllipticPi}\left(\sqrt{\frac{\left(-1-I\sqrt{3}\right)x}{\left(1-I\sqrt{3}\right)(-2+x)}}, \frac{1-I\sqrt{3}}{-1-I\sqrt{3}}, \sqrt{\frac{\left(1-I\sqrt{3}\right)\left(I\sqrt{3}-1\right)}{\left(-1-I\sqrt{3}\right)\left(I\sqrt{3}+1\right)}}\right)\right)\right)$$

$$- \frac{1}{432\sqrt{-x\,(-2+x)\,\left(x-I\sqrt{3}-1\right)\,\left(x-1+I\sqrt{3}\right)}} \left(7\left(x\left(x-I\sqrt{3}-1\right)\left(x-1+I\sqrt{3}\right)+2\left(I\sqrt{3}-1\right)\sqrt{\frac{\left(-1-I\sqrt{3}\right)x}{\left(1-I\sqrt{3}\right)(-2+x)}}\,(-2\right.\right.$$

$$\left.+x)^{2}\sqrt{\frac{x-I\sqrt{3}-1}{\left(I\sqrt{3}+1\right)(-2+x)}}\sqrt{\frac{x-1+I\sqrt{3}}{\left(1-I\sqrt{3}\right)(-2+x)}}\left(\frac{\left(6-2I\sqrt{3}\right)\mathrm{EllipticF}\left(\sqrt{\frac{\left(-1-I\sqrt{3}\right)x}{\left(1-I\sqrt{3}\right)(-2+x)}},\sqrt{\frac{\left(1-I\sqrt{3}\right)\left(I\sqrt{3}-1\right)}{\left(-1-I\sqrt{3}\right)\left(I\sqrt{3}+1\right)}}\right)}{2\left(-1-I\sqrt{3}\right)}\right.$$

$$+\frac{\left(-1-I\sqrt{3}\right)\mathrm{EllipticE}\left(\sqrt{\frac{\left(-1-I\sqrt{3}\right)x}{\left(1-I\sqrt{3}\right)(-2+x)}},\sqrt{\frac{\left(1-I\sqrt{3}\right)\left(I\sqrt{3}-1\right)}{\left(-1-I\sqrt{3}\right)\left(I\sqrt{3}+1\right)}}\right)}{2}$$

$$\left.\left.\left.-\frac{4\,\mathrm{EllipticPi}\left(\sqrt{\frac{\left(-1-I\sqrt{3}\right)x}{\left(1-I\sqrt{3}\right)(-2+x)}},\frac{I\sqrt{3}-1}{I\sqrt{3}+1},\sqrt{\frac{\left(1-I\sqrt{3}\right)\left(I\sqrt{3}-1\right)}{\left(-1-I\sqrt{3}\right)\left(I\sqrt{3}+1\right)}}\right)}{-1-I\sqrt{3}}\right)\right)\right)$$

Problem 212: Result more than twice size of optimal antiderivative.

$$\int\frac{1}{\left((2-x)\,x\,(x^2-2x+4)\right)^{3/2}}\,dx$$

Optimal(type 4, 61 leaves, 6 steps):

$$-\frac{\mathrm{EllipticE}\left(-1+x,\frac{I}{3}\sqrt{3}\right)\sqrt{3}}{24}+\frac{\mathrm{EllipticF}\left(-1+x,\frac{I}{3}\sqrt{3}\right)\sqrt{3}}{12}+\frac{\left(5+(-1+x)^2\right)(-1+x)}{24\sqrt{3-2\,(-1+x)^2-(-1+x)^4}}$$

Result(type 4, 962 leaves):

$$-\frac{-x^3+4x^2-8x+8}{32\sqrt{x\,(-x^3+4x^2-8x+8)}}+\frac{2x\left(\frac{1}{24}+\frac{x^2}{192}\right)}{\sqrt{-x\,(x^3-4x^2+8x-8)}}+\frac{1}{6\left(-1-I\sqrt{3}\right)\sqrt{-x\,(-2+x)\left(x-I\sqrt{3}-1\right)\left(x-1+I\sqrt{3}\right)}}\left(\left(I\sqrt{3}\right.\right.$$

$$\left.-1\right)\sqrt{\frac{\left(-1-I\sqrt{3}\right)x}{\left(1-I\sqrt{3}\right)(-2+x)}}\,(-2+x)^2\sqrt{\frac{x-I\sqrt{3}-1}{\left(I\sqrt{3}+1\right)(-2+x)}}\sqrt{\frac{x-1+I\sqrt{3}}{\left(1-I\sqrt{3}\right)(-2+x)}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(-1-I\sqrt{3}\right)x}{\left(1-I\sqrt{3}\right)(-2+x)}},\right.$$

$$\left.\left.\sqrt{\frac{\left(1-I\sqrt{3}\right)\left(I\sqrt{3}-1\right)}{\left(-1-I\sqrt{3}\right)\left(I\sqrt{3}+1\right)}}\right)\right)+\frac{1}{6\left(-1-I\sqrt{3}\right)\sqrt{-x\,(-2+x)\left(x-I\sqrt{3}-1\right)\left(x-1+I\sqrt{3}\right)}}\left(\left(I\sqrt{3}\right.\right.$$

$$\left.-1\right)\sqrt{\frac{\left(-1-I\sqrt{3}\right)x}{\left(1-I\sqrt{3}\right)(-2+x)}}\,(-2+x)^{2}\sqrt{\frac{x-I\sqrt{3}-1}{\left(I\sqrt{3}+1\right)(-2+x)}}\sqrt{\frac{x-1+I\sqrt{3}}{\left(1-I\sqrt{3}\right)(-2+x)}}\left(2\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(-1-I\sqrt{3}\right)x}{\left(1-I\sqrt{3}\right)(-2+x)}},\right.\right.$$

$$\sqrt{\frac{(1-I\sqrt{3})(I\sqrt{3}-1)}{(-1-I\sqrt{3})(I\sqrt{3}+1)}}\Bigg) - 2\,\text{EllipticPi}\left(\sqrt{\frac{(-1-I\sqrt{3})x}{(1-I\sqrt{3})(-2+x)}}, \frac{1-I\sqrt{3}}{-1-I\sqrt{3}}, \sqrt{\frac{(1-I\sqrt{3})(I\sqrt{3}-1)}{(-1-I\sqrt{3})(I\sqrt{3}+1)}}\right)\Bigg)\Bigg)$$

$$- \frac{1}{24\sqrt{-x(-2+x)\,(x-I\sqrt{3}-1)\,(x-1+I\sqrt{3})}}\Bigg(x\,(x-I\sqrt{3}-1)\,(x-1+I\sqrt{3}) + 2\,(I\sqrt{3}-1)\sqrt{\frac{(-1-I\sqrt{3})x}{(1-I\sqrt{3})(-2+x)}}\,(-2$$

$$+x)^2\sqrt{\frac{x-I\sqrt{3}-1}{(I\sqrt{3}+1)(-2+x)}}\sqrt{\frac{x-1+I\sqrt{3}}{(1-I\sqrt{3})(-2+x)}}\Bigg(\frac{(6-2I\sqrt{3})\,\text{EllipticF}\left(\sqrt{\frac{(-1-I\sqrt{3})x}{(1-I\sqrt{3})(-2+x)}}, \sqrt{\frac{(1-I\sqrt{3})(I\sqrt{3}-1)}{(-1-I\sqrt{3})(I\sqrt{3}+1)}}\right)}{2\,(-1-I\sqrt{3})}$$

$$+ \frac{(-1-I\sqrt{3})\,\text{EllipticE}\left(\sqrt{\frac{(-1-I\sqrt{3})x}{(1-I\sqrt{3})(-2+x)}}, \sqrt{\frac{(1-I\sqrt{3})(I\sqrt{3}-1)}{(-1-I\sqrt{3})(I\sqrt{3}+1)}}\right)}{2}$$

$$- \frac{4\,\text{EllipticPi}\left(\sqrt{\frac{(-1-I\sqrt{3})x}{(1-I\sqrt{3})(-2+x)}}, \frac{I\sqrt{3}-1}{I\sqrt{3}+1}, \sqrt{\frac{(1-I\sqrt{3})(I\sqrt{3}-1)}{(-1-I\sqrt{3})(I\sqrt{3}+1)}}\right)}{-1-I\sqrt{3}}\Bigg)\Bigg)$$

Problem 213: Result more than twice size of optimal antiderivative.

$$\int \sqrt{d^2 x^4 + 4\,c\,d\,x^3 + 4\,x^2 c^2 + 4\,a\,c}\;dx$$

Optimal(type 4, 668 leaves, 5 steps):

$$\frac{\left(\frac{c}{d}+x\right)\sqrt{d^2x^4+4\,c\,d\,x^3+4\,x^2c^2+4\,a\,c}}{3} - \frac{2\,c^2\left(\frac{c}{d}+x\right)\sqrt{d^2x^4+4\,c\,d\,x^3+4\,x^2c^2+4\,a\,c}}{3\left(\sqrt{c}+\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4\,a\,d^2+c^3}}\right)\sqrt{4\,a\,d^2+c^3}}$$

$$+ \frac{1}{3\cos\left(2\arctan\left(\frac{d\,x+c}{c^{1/4}\,(4\,a\,d^2+c^3)^{1/4}}\right)\right)d^3\sqrt{d^2x^4+4\,c\,d\,x^3+4\,x^2c^2+4\,a\,c}}\Bigg(2\,c^{9/4}\,(4\,a\,d^2+c^3)^{3/}$$

$$4\sqrt{\cos\left(2\arctan\left(\frac{dx+c}{c^{1/4}\left(4ad^2+c^3\right)^{1/4}}\right)\right)^2}\,\mathrm{EllipticE}\left(\sin\left(2\arctan\left(\frac{dx+c}{c^{1/4}\left(4ad^2+c^3\right)^{1/4}}\right)\right),\frac{\sqrt{2+\frac{2c^{3/2}}{\sqrt{4ad^2+c^3}}}}{2}\right)\left(\sqrt{c}\right.$$

$$\left.+\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}\right)\sqrt{\frac{d^2\left(d^2x^4+4cdx^3+4x^2c^2+4ac\right)}{\left(4ad^2+c^3\right)\left(\sqrt{c}+\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}\right)^2}}\Bigg)$$

$$+\frac{1}{3\cos\left(2\arctan\left(\frac{dx+c}{c^{1/4}\left(4ad^2+c^3\right)^{1/4}}\right)\right)d^3\sqrt{d^2x^4+4cdx^3+4x^2c^2+4ac}}\Bigg(c^{3/4}\left(4ad^2+c^3\right)^{1/}$$

$$4\sqrt{\cos\left(2\arctan\left(\frac{dx+c}{c^{1/4}\left(4ad^2+c^3\right)^{1/4}}\right)\right)^2}\,\mathrm{EllipticF}\left(\sin\left(2\arctan\left(\frac{dx+c}{c^{1/4}\left(4ad^2+c^3\right)^{1/4}}\right)\right),\frac{\sqrt{2+\frac{2c^{3/2}}{\sqrt{4ad^2+c^3}}}}{2}\right)\left(\sqrt{c}+\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}\right)\left(c^3\right.$$

$$\left.+4ad^2-c^{3/2}\sqrt{4ad^2+c^3}\right)\sqrt{\frac{d^2\left(d^2x^4+4cdx^3+4x^2c^2+4ac\right)}{\left(4ad^2+c^3\right)\left(\sqrt{c}+\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}\right)^2}}\Bigg)$$

Result(type ?, 4889 leaves): Display of huge result suppressed!

Problem 214: Result more than twice size of optimal antiderivative.

$$\int\sqrt{8e^3x^4+8de^2x^3-d^3x+8ae^2}\,dx$$

Optimal(type 4, 715 leaves, 5 steps):

$$\frac{\left(\frac{d}{4e}+x\right)\sqrt{8e^3x^4+8de^2x^3-d^3x+8ae^2}}{3}-\frac{2d^2\left(\frac{d}{4e}+x\right)\sqrt{8e^3x^4+8de^2x^3-d^3x+8ae^2}}{\left(1+\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{256ae^3+5d^4}}\right)\sqrt{256ae^3+5d^4}}$$

$$+\frac{1}{16\cos\left(2\arctan\left(\frac{4ex+d}{(256ae^3+5d^4)^{1/4}}\right)\right)e^2\sqrt{8e^3x^4+8de^2x^3-d^3x+8ae^2}}\Bigg(d^2\,(256ae^3+5d^4)^{3/}$$

$$4\sqrt{\cos\left(2\arctan\left(\frac{4ex+d}{(256ae^3+5d^4)^{1/4}}\right)\right)^2}\ \mathrm{EllipticE}\left(\sin\left(2\arctan\left(\frac{4ex+d}{(256ae^3+5d^4)^{1/4}}\right)\right),\frac{\sqrt{2+\frac{6d^2}{\sqrt{256ae^3+5d^4}}}}{2}\right)\Bigg(1$$

$$+\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{256ae^3+5d^4}}\Bigg)\sqrt{\frac{e\,(8e^3x^4+8de^2x^3-d^3x+8ae^2)}{(256ae^3+5d^4)\left(1+\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{256ae^3+5d^4}}\right)^2}}\ \sqrt{2}\Bigg)$$

$$+\frac{1}{96\cos\left(2\arctan\left(\frac{4ex+d}{(256ae^3+5d^4)^{1/4}}\right)\right)e^2\sqrt{8e^3x^4+8de^2x^3-d^3x+8ae^2}}\Bigg((256ae^3+5d^4)^{1/}$$

$$4\sqrt{\cos\left(2\arctan\left(\frac{4ex+d}{(256ae^3+5d^4)^{1/4}}\right)\right)^2}\ \mathrm{EllipticF}\left(\sin\left(2\arctan\left(\frac{4ex+d}{(256ae^3+5d^4)^{1/4}}\right)\right),\frac{\sqrt{2+\frac{6d^2}{\sqrt{256ae^3+5d^4}}}}{2}\right)\Bigg(1$$

$$+\frac{16 e^2 \left(\frac{d}{4 e}+x\right)^2}{\sqrt{256 a e^3+5 d^4}}\Bigg) \left(5 d^4+256 a e^3-3 d^2 \sqrt{256 a e^3+5 d^4}\right) \sqrt{\frac{e \left(8 e^3 x^4+8 d e^2 x^3-d^3 x+8 a e^2\right)}{\left(256 a e^3+5 d^4\right) \left(1+\frac{16 e^2 \left(\frac{d}{4 e}+x\right)^2}{\sqrt{256 a e^3+5 d^4}}\right)^2}}\ \sqrt{2}\Bigg)$$

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Result(type ?, 7886 leaves): Display of huge result suppressed!

Problem 215: Result more than twice size of optimal antiderivative.
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$$\int \sqrt{-x^4+4 x^3-8 x^2+a+8 x}\, dx$$

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Optimal(type 4, 455 leaves, 7 steps):
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$$-\frac{2 (-1+x) \left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right) \left(1-\sqrt{4+a}\right)}{3 \sqrt{3+a-2 (-1+x)^2-(-1+x)^4}}+\frac{(-1+x) \sqrt{3+a-2 (-1+x)^2-(-1+x)^4}}{3}$$

$$+\frac{1}{3 \sqrt{3+a-2 (-1+x)^2-(-1+x)^4}\ \sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}}}\Bigg(2 (3$$

$$+a) \sqrt{\frac{1}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}}\ \sqrt{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}\ \mathrm{EllipticF}\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}\ \sqrt{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}}, \sqrt{-\frac{2 \sqrt{4+a}}{1-\sqrt{4+a}}}\right) \Bigg(1$$

$$+\frac{(-1+x)^2}{1-\sqrt{4+a}}\Bigg) \sqrt{1+\sqrt{4+a}}\Bigg)$$

$$+\frac{1}{3 \sqrt{3+a-2 (-1+x)^2-(-1+x)^4}\ \sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}}}\Bigg(2 \sqrt{\frac{1}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}}\ \sqrt{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}\ \mathrm{EllipticE}\Bigg(1 \Bigg/$$

$$\left(\sqrt{1+\sqrt{4+a}}\,\sqrt{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}\right)(-1+x),\sqrt{-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}}\right)\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)\left(1-\sqrt{4+a}\right)\sqrt{1+\sqrt{4+a}}\right)$$

Result(type ?, 2518 leaves): Display of huge result suppressed!

Problem 216: Result more than twice size of optimal antiderivative.

$$\int x\sqrt{-x^4+4x^3-8x^2+a+8x}\,dx$$

Optimal(type 4, 516 leaves, 12 steps):

$$\frac{(4+a)\arctan\left(\frac{1+(-1+x)^2}{\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}\right)}{4}-\frac{2(-1+x)\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)\left(1-\sqrt{4+a}\right)}{3\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$
$$+\frac{\left(1+(-1+x)^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}{4}+\frac{(-1+x)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}{3}$$
$$+\frac{1}{3\sqrt{3+a-2(-1+x)^2-(-1+x)^4}\sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}}}\left(2(3\right.$$
$$+a)\sqrt{\frac{1}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}}\sqrt{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}\,\mathrm{EllipticF}\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}\sqrt{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}},\sqrt{-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}}\right)\left(1\right.$$
$$\left.\left.+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)\sqrt{1+\sqrt{4+a}}\right)$$
$$+\frac{1}{3\sqrt{3+a-2(-1+x)^2-(-1+x)^4}\sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}}}\left(2\sqrt{\frac{1}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}}\sqrt{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}\,\mathrm{EllipticE}\left(1\right/\right.$$

$$\left(\sqrt{1+\sqrt{4+a}}\,\sqrt{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}\right)(-1+x),\ \sqrt{-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}}\right)\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)\left(1-\sqrt{4+a}\right)\sqrt{1+\sqrt{4+a}}\right)$$

Result(type ?, 2550 leaves): Display of huge result suppressed!

Problem 217: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{8x^4-x^3+8x+8}}\,dx$$

Optimal(type 4, 155 leaves, 4 steps):

$$-\frac{1}{696\cos\left(2\arctan\left(\frac{(4+x)\,29^{3/4}\sqrt{3}}{87x}\right)\right)\sqrt{8x^4-x^3+8x+8}}\left(x^2\sqrt{\cos\left(2\arctan\left(\frac{(4+x)\,29^{3/4}\sqrt{3}}{87x}\right)\right)^2}\ \mathrm{EllipticF}\left(\sin\left(2\arctan\left(\frac{1}{87x}\left((4+x)\,29^{3/4}\sqrt{3}\right)\right)\right),\ \frac{\sqrt{1682+58\sqrt{29}}}{58}\right)\left(87+\frac{(4+x)^2\sqrt{29}}{x^2}\right)\sqrt{\frac{261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4}{\left(87+\frac{(4+x)^2\sqrt{29}}{x^2}\right)^2}}\ 29^{3/4}\sqrt{3}\right)$$

Result(type 4, 964 leaves):

$$\Bigg((\mathit{RootOf}(8\_Z^4-\_Z^3+8\_Z+8,\mathit{index}=1)-\mathit{RootOf}(8\_Z^4-\_Z^3+8\_Z+8,\mathit{index}=4))$$

$$\sqrt{\frac{(\mathit{RootOf}(8\_Z^4-\_Z^3+8\_Z+8,\mathit{index}=4)-\mathit{RootOf}(8\_Z^4-\_Z^3+8\_Z+8,\mathit{index}=2))\,(x-\mathit{RootOf}(8\_Z^4-\_Z^3+8\_Z+8,\mathit{index}=1))}{(\mathit{RootOf}(8\_Z^4-\_Z^3+8\_Z+8,\mathit{index}=4)-\mathit{RootOf}(8\_Z^4-\_Z^3+8\_Z+8,\mathit{index}=1))\,(x-\mathit{RootOf}(8\_Z^4-\_Z^3+8\_Z+8,\mathit{index}=2))}}\,(x-\mathit{RootOf}(8\_Z^4-\_Z^3+8\_Z+8,\mathit{index}=2))^2$$

$$\sqrt{\frac{(\mathit{RootOf}(8\_Z^4-\_Z^3+8\_Z+8,\mathit{index}=2)-\mathit{RootOf}(8\_Z^4-\_Z^3+8\_Z+8,\mathit{index}=1))\,(x-\mathit{RootOf}(8\_Z^4-\_Z^3+8\_Z+8,\mathit{index}=3))}{(\mathit{RootOf}(8\_Z^4-\_Z^3+8\_Z+8,\mathit{index}=3)-\mathit{RootOf}(8\_Z^4-\_Z^3+8\_Z+8,\mathit{index}=1))\,(x-\mathit{RootOf}(8\_Z^4-\_Z^3+8\_Z+8,\mathit{index}=2))}}$$

$$\sqrt{\frac{(\mathit{RootOf}(8\_Z^4-\_Z^3+8\_Z+8,\mathit{index}=2)-\mathit{RootOf}(8\_Z^4-\_Z^3+8\_Z+8,\mathit{index}=1))\,(x-\mathit{RootOf}(8\_Z^4-\_Z^3+8\_Z+8,\mathit{index}=4))}{(\mathit{RootOf}(8\_Z^4-\_Z^3+8\_Z+8,\mathit{index}=4)-\mathit{RootOf}(8\_Z^4-\_Z^3+8\_Z+8,\mathit{index}=1))\,(x-\mathit{RootOf}(8\_Z^4-\_Z^3+8\_Z+8,\mathit{index}=2))}}\,\sqrt{2}$$

$$\mathrm{EllipticF}\Bigg($$

$$\sqrt{\frac{(RootOf(8\_Z^4-\_Z^3+8\_Z+8, index=4)-RootOf(8\_Z^4-\_Z^3+8\_Z+8, index=2))(x-RootOf(8\_Z^4-\_Z^3+8\_Z+8, index=1))}{(RootOf(8\_Z^4-\_Z^3+8\_Z+8, index=4)-RootOf(8\_Z^4-\_Z^3+8\_Z+8, index=1))(x-RootOf(8\_Z^4-\_Z^3+8\_Z+8, index=2))}},$$

$$(((RootOf(8\_Z^4-\_Z^3+8\_Z+8, index=2)-RootOf(8\_Z^4-\_Z^3+8\_Z+8, index=3))(RootOf(8\_Z^4-\_Z^3+8\_Z+8, index=1)$$
$$-RootOf(8\_Z^4-\_Z^3+8\_Z+8, index=4)))\Big/((RootOf(8\_Z^4-\_Z^3+8\_Z+8, index=1)-RootOf(8\_Z^4-\_Z^3+8\_Z+8, index$$
$$=3))(RootOf(8\_Z^4-\_Z^3+8\_Z+8, index=2)-RootOf(8\_Z^4-\_Z^3+8\_Z+8, index=4))))^{1/2}))\Big/\Big(2(RootOf(8\_Z^4-\_Z^3+8\_Z+8, index$$
$$=4)-RootOf(8\_Z^4-\_Z^3+8\_Z+8, index=2))(RootOf(8\_Z^4-\_Z^3+8\_Z+8, index=2)-RootOf(8\_Z^4-\_Z^3+8\_Z+8, index$$
$$=1))$$
$$((x-RootOf(8\_Z^4-\_Z^3+8\_Z+8, index=1))(x-RootOf(8\_Z^4-\_Z^3+8\_Z+8, index=2))(x-RootOf(8\_Z^4-\_Z^3+8\_Z+8, index=3))(x-RootOf$$
$$=4)))^{1/2}\Big)$$

Problem 218: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{3x^4+15x^3-44x^2-6x+9}}\,dx$$

Optimal(type 4, 156 leaves, 4 steps):

$$-\frac{1}{7356\cos\left(2\arctan\left(\frac{(6-x)\,613^{3/4}}{613\,x}\right)\right)\sqrt{3x^4+15x^3-44x^2-6x+9}}\left(x^2\sqrt{\cos\left(2\arctan\left(\frac{(6-x)\,613^{3/4}}{613\,x}\right)\right)^2}\,\mathrm{EllipticF}\left(\sin\left(2\arctan\left(\frac{1}{613\,x}((6-x)\,613^{3/4})\right)\right),\frac{\sqrt{751538+111566\sqrt{613}}}{1226}\right)\left(\frac{(6-x)^2}{x^2}+\sqrt{613}\right)\sqrt{\frac{613-182\left(1-\frac{6}{x}\right)^2+\left(-1+\frac{6}{x}\right)^4}{\left(\frac{(6-x)^2}{x^2}+\sqrt{613}\right)^2}}\,613^{3/4}\right)$$

Result(type 4, 1181 leaves):

$$\Big(2(-RootOf(3\_Z^4+15\_Z^3-44\_Z^2-6\_Z+9, index=4)+RootOf(3\_Z^4+15\_Z^3-44\_Z^2-6\_Z+9, index$$
$$=1))$$
$$(((x-RootOf(3\_Z^4+15\_Z^3-44\_Z^2-6\_Z+9, index=1))(-RootOf(3\_Z^4+15\_Z^3-44\_Z^2-6\_Z+9, index=4)+RootOf(3\_Z^4+15\_Z^3-44\_Z^2-6\_$$

$+ RootOf(3\_Z^4 + 15\_Z^3 - 44\_Z^2 - 6\_Z + 9, index = 1)) (x - RootOf(3\_Z^4 + 15\_Z^3 - 44\_Z^2 - 6\_Z + 9, index = 2))))^{1/2} (x - RootOf(3\_Z^4$

$+ 15\_Z^3 - 44\_Z^2 - 6\_Z + 9, index = 2))$

${}^2 (-((x - RootOf(3\_Z^4 + 15\_Z^3 - 44\_Z^2 - 6\_Z + 9, index = 3)) (RootOf(3\_Z^4 + 15\_Z^3 - 44\_Z^2 - 6\_Z + 9, index = 2) - RootOf(3\_Z^4 + 15\_Z^3 - 44\_Z^2 - 6$

$= 1)))/((-RootOf(3\_Z^4 + 15\_Z^3 - 44\_Z^2 - 6\_Z + 9, index = 3) + RootOf(3\_Z^4 + 15\_Z^3 - 44\_Z^2 - 6\_Z + 9, index = 1)) (x - RootOf(3\_Z^4 + 15\_Z^3$

$- 44\_Z^2 - 6\_Z + 9, index = 2))))$

${}^{1/2} (-((x - RootOf(3\_Z^4 + 15\_Z^3 - 44\_Z^2 - 6\_Z + 9, index = 4)) (RootOf(3\_Z^4 + 15\_Z^3 - 44\_Z^2 - 6\_Z + 9, index = 2) - RootOf(3\_Z^4 + 15\_Z^3 - 44\_Z^2 -$

$= 1)))/((-RootOf(3\_Z^4 + 15\_Z^3 - 44\_Z^2 - 6\_Z + 9, index = 4) + RootOf(3\_Z^4 + 15\_Z^3 - 44\_Z^2 - 6\_Z + 9, index = 1)) (x - RootOf(3\_Z^4 + 15\_Z^3$

$- 44\_Z^2 - 6\_Z + 9, index = 2))))$

${}^{1/2} \sqrt{3}$

$\mathrm{EllipticF}\Bigg((((x - RootOf(3\_Z^4 + 15\_Z^3 - 44\_Z^2 - 6\_Z + 9, index = 1)) (-RootOf(3\_Z^4 + 15\_Z^3 - 44\_Z^2 - 6\_Z + 9, index = 4)$

$+ RootOf(3\_Z^4 + 15\_Z^3 - 44\_Z^2 - 6\_Z + 9, index = 2)))/((-RootOf(3\_Z^4 + 15\_Z^3 - 44\_Z^2 - 6\_Z + 9, index = 4) + RootOf(3\_Z^4 + 15\_Z^3 - 44\_Z^2$

$- 6\_Z + 9, index = 1)) (x - RootOf(3\_Z^4 + 15\_Z^3 - 44\_Z^2 - 6\_Z + 9, index = 2))))^{1/2},$

$(((RootOf(3\_Z^4 + 15\_Z^3 - 44\_Z^2 - 6\_Z + 9, index = 2) - RootOf(3\_Z^4 + 15\_Z^3 - 44\_Z^2 - 6\_Z + 9, index = 3)) (-RootOf(3\_Z^4 + 15\_Z^3 - 44\_Z^2 - 6\_Z +$

$- 44\_Z^2 - 6\_Z + 9, index = 4) + RootOf(3\_Z^4 + 15\_Z^3 - 44\_Z^2 - 6\_Z + 9, index = 2))))^{1/2}\Big)\Big) \Big/ \Big(3 (RootOf(3\_Z^4 + 15\_Z^3 - 44\_Z^2 - 6\_Z + 9,$

$index = 4) - RootOf(3\_Z^4 + 15\_Z^3 - 44\_Z^2 - 6\_Z + 9, index = 2)) (RootOf(3\_Z^4 + 15\_Z^3 - 44\_Z^2 - 6\_Z + 9, index = 2) - RootOf(3\_Z^4 + 15\_Z^3$

$- 44\_Z^2 - 6\_Z + 9, index = 1))$

$$\left(\left(x-RootOf\left(3\_Z^4+15\_Z^3-44\_Z^2-6\_Z+9, index=1\right)\right)\left(x-RootOf\left(3\_Z^4+15\_Z^3-44\_Z^2-6\_Z+9, index=2\right)\right)\left(x-RootOf\left(3\_Z^4+15\_Z^3-44\_Z^2\right.\right.\right.$$
$$\left.\left.\left.=4\right)\right)^{1/2}\right)$$

Problem 223: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{-3x^2+3+(-5-4x)\sqrt{-x^2+1}}\,dx$$

Optimal(type 2, 27 leaves, 16 steps):

$$\frac{3}{5(4+5x)}+\frac{\sqrt{-x^2+1}}{4+5x}$$

Result(type 2, 80 leaves):

$$\frac{3}{5(4+5x)}+\frac{5\left(-\left(x+\frac{4}{5}\right)^2+\frac{8x}{5}+\frac{41}{25}\right)^{3/2}}{9\left(x+\frac{4}{5}\right)}+\frac{5x\sqrt{-\left(x+\frac{4}{5}\right)^2+\frac{8x}{5}+\frac{41}{25}}}{9}+\frac{\sqrt{-(-1+x)^2-2x+2}}{18}-\frac{\sqrt{-(1+x)^2+2x+2}}{2}$$

Problem 228: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{2-3x}\,\sqrt{2+3x}}\,dx$$

Optimal(type 3, 6 leaves, 2 steps):

$$\frac{\arcsin\left(\frac{3x}{2}\right)}{3}$$

Result(type 3, 33 leaves):

$$\frac{\sqrt{(2-3x)(2+3x)}\,\arcsin\left(\frac{3x}{2}\right)}{3\sqrt{2-3x}\,\sqrt{2+3x}}$$

Problem 229: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{3-x}\,\sqrt{5+x}}\,dx$$

Optimal(type 3, 6 leaves, 3 steps):

$$\arcsin\left(\frac{1}{4}+\frac{x}{4}\right)$$

Result(type 3, 30 leaves):

$$\frac{\sqrt{(3-x)\,(5+x)}\,\arcsin\left(\frac{1}{4}+\frac{x}{4}\right)}{\sqrt{3-x}\,\sqrt{5+x}}$$

Problem 231: Result more than twice size of optimal antiderivative.

$$\int \frac{(a x+1)^{3/2}}{x\,(-a x+1)^{3/2}}\,dx$$

Optimal(type 3, 43 leaves, 7 steps):

$$-\arcsin(a x)-\operatorname{arctanh}\left(\sqrt{-a x+1}\,\sqrt{a x+1}\right)+\frac{4\sqrt{a x+1}}{\sqrt{-a x+1}}$$

Result(type 3, 129 leaves):

$$\frac{1}{(a x-1)\sqrt{-a^2 x^2+1}}\left(\left(-\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2 x^2+1}}\right)\operatorname{csgn}(a)\,x a-\arctan\left(\frac{\operatorname{csgn}(a)\,a x}{\sqrt{-a^2 x^2+1}}\right)x a+\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2 x^2+1}}\right)\operatorname{csgn}(a)-4\sqrt{-a^2 x^2+1}\,\operatorname{csgn}(a)\right.\right.$$
$$\left.\left.+\arctan\left(\frac{\operatorname{csgn}(a)\,a x}{\sqrt{-a^2 x^2+1}}\right)\right)\operatorname{csgn}(a)\sqrt{-a x+1}\,\sqrt{a x+1}\right)$$

Problem 233: Unable to integrate problem.

$$\int \left(\frac{c x^2+a+b}{d}\right)^m dx$$

Optimal(type 5, 47 leaves, 3 steps):

$$\frac{d x\left(\frac{a+b}{d}+\frac{c x^2}{d}\right)^{1+m}\operatorname{hypergeom}\left(\left[1,\frac{3}{2}+m\right],\left[\frac{3}{2}\right],-\frac{c x^2}{a+b}\right)}{a+b}$$

Result(type 8, 16 leaves):

$$\int \left(\frac{c x^2+a+b}{d}\right)^m dx$$

Problem 234: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x-\sqrt{-x^2+1}}\,dx$$

Optimal(type 3, 29 leaves, 7 steps):

$$-\frac{\arcsin(x)}{2}-\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{-x^2+1}}\right)}{2}+\frac{\ln(-2 x^2+1)}{4}$$

Result(type 3, 174 leaves):

$$\frac{\ln(2x^2-1)}{4}+\frac{\sqrt{2}\,\sqrt{-4\left(x-\frac{\sqrt{2}}{2}\right)^2-4\left(x-\frac{\sqrt{2}}{2}\right)\sqrt{2}+2}}{8}-\frac{\arcsin(x)}{2}-\frac{\operatorname{arctanh}\left(\frac{\left(-\left(x-\frac{\sqrt{2}}{2}\right)\sqrt{2}+1\right)\sqrt{2}}{\sqrt{-4\left(x-\frac{\sqrt{2}}{2}\right)^2-4\left(x-\frac{\sqrt{2}}{2}\right)\sqrt{2}+2}}\right)}{4}$$
$$-\frac{\sqrt{2}\,\sqrt{-4\left(x+\frac{\sqrt{2}}{2}\right)^2+4\left(x+\frac{\sqrt{2}}{2}\right)\sqrt{2}+2}}{8}+\frac{\operatorname{arctanh}\left(\frac{\left(\left(x+\frac{\sqrt{2}}{2}\right)\sqrt{2}+1\right)\sqrt{2}}{\sqrt{-4\left(x+\frac{\sqrt{2}}{2}\right)^2+4\left(x+\frac{\sqrt{2}}{2}\right)\sqrt{2}+2}}\right)}{4}$$

Problem 236: Result more than twice size of optimal antiderivative.

$$\int \frac{x\sqrt{-x^2+2}}{x-\sqrt{-x^2+2}}\,dx$$

Optimal(type 3, 46 leaves, 12 steps):

$$-\frac{x^2}{4}-\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{-x^2+2}}\right)}{2}+\frac{\ln(1-x)}{4}+\frac{\ln(1+x)}{4}+\frac{x\sqrt{-x^2+2}}{4}$$

Result(type 3, 110 leaves):

$$-\frac{x^2}{4}+\frac{\ln(-1+x)}{4}+\frac{\ln(1+x)}{4}+\frac{x\sqrt{-x^2+2}}{4}+\frac{\sqrt{-(-1+x)^2-2x+3}}{4}-\frac{\operatorname{arctanh}\left(\frac{-2x+4}{2\sqrt{-(-1+x)^2-2x+3}}\right)}{4}-\frac{\sqrt{-(1+x)^2+2x+3}}{4}$$
$$+\frac{\operatorname{arctanh}\left(\frac{2x+4}{2\sqrt{-(1+x)^2+2x+3}}\right)}{4}$$

Problem 237: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\,\sqrt{d x^2+c}}\,dx$$

Optimal(type 3, 54 leaves, 5 steps):

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{d}\,\sqrt{a x^2+b}}{\sqrt{a}\,\sqrt{d x^2+c}}\right)\sqrt{a x^2+b}}{x\sqrt{a}\,\sqrt{d}\,\sqrt{a+\frac{b}{x^2}}}$$

Result(type 3, 116 leaves):

$$\frac{(a x^2+b)\ln\left(\frac{2 a d x^2+2\sqrt{a d x^4+a c x^2+b d x^2+b c}\,\sqrt{a d}+a c+b d}{2\sqrt{a d}}\right)\sqrt{d x^2+c}}{2\sqrt{\frac{a x^2+b}{x^2}}\,x\sqrt{a d x^4+a c x^2+b d x^2+b c}\,\sqrt{a d}}$$

Problem 240: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{2-\frac{b}{x^2}}}{2 x^2-b}\,dx$$

Optimal(type 3, 14 leaves, 3 steps):

$$-\frac{\operatorname{arccsc}\left(\frac{x\sqrt{2}}{\sqrt{b}}\right)}{\sqrt{b}}$$

Result(type 3, 61 leaves):

$$-\frac{\sqrt{\frac{2 x^2-b}{x^2}}\,x\ln\left(\frac{2\left(\sqrt{-b}\,\sqrt{2 x^2-b}-b\right)}{x}\right)}{\sqrt{2 x^2-b}\,\sqrt{-b}}$$

Problem 241: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}{e x+d}\,dx$$

Optimal(type 3, 157 leaves, 10 steps):

$$\frac{\operatorname{arctanh}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)\sqrt{a}}{e}-\frac{\operatorname{arctanh}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)\sqrt{c}}{d}$$

$$-\frac{\operatorname{arctanh}\left(\frac{2ad-be+\frac{bd-2ec}{x}}{2\sqrt{ad^2-e(bd-ec)}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)\sqrt{ad^2-e(bd-ec)}}{de}$$

Result(type 3, 382 leaves):

$$\frac{1}{\sqrt{ax^2+bx+c}\,d\,e^2\sqrt{\frac{ad^2-bde+ce^2}{e^2}}}\left(\sqrt{\frac{ax^2+bx+c}{x^2}}\,x\left(\ln\left(\frac{2\sqrt{ax^2+bx+c}\sqrt{a}+2ax+b}{2\sqrt{a}}\right)\sqrt{a}\,d\,e\sqrt{\frac{ad^2-bde+ce^2}{e^2}}\right.\right.$$

$$-\sqrt{c}\ln\left(\frac{2c+bx+2\sqrt{c}\sqrt{ax^2+bx+c}}{x}\right)e^2\sqrt{\frac{ad^2-bde+ce^2}{e^2}}$$

$$+d^2\ln\left(\frac{2\sqrt{ax^2+bx+c}\sqrt{\frac{ad^2-bde+ce^2}{e^2}}\,e-2adx+xbe-bd+2ec}{ex+d}\right)a$$

$$-\ln\left(\frac{2\sqrt{ax^2+bx+c}\sqrt{\frac{ad^2-bde+ce^2}{e^2}}\,e-2adx+xbe-bd+2ec}{ex+d}\right)bde$$

$$\left.\left.+\ln\left(\frac{2\sqrt{ax^2+bx+c}\sqrt{\frac{ad^2-bde+ce^2}{e^2}}\,e-2adx+xbe-bd+2ec}{ex+d}\right)ce^2\right)\right)$$

Problem 244: Unable to integrate problem.

$$\int \frac{x^{-1+m}\left(2am+b(2m-n)x^n\right)}{2\left(a+bx^n\right)^{3/2}}\,dx$$

Optimal(type 3, 13 leaves, 2 steps):

$$\frac{x^m}{\sqrt{a+bx^n}}$$

Result(type 8, 35 leaves):

$$\int \frac{x^{-1+m} \left(2 a m + b \left(2 m - n\right) x^{n}\right)}{2 \left(a + b x^{n}\right)^{3/2}} \, dx$$

Problem 252: Result more than twice size of optimal antiderivative.

$$\int \sqrt{\frac{x}{1+x}} \, dx$$

Optimal(type 3, 16 leaves, 4 steps):

$$-\operatorname{arcsinh}\left(\sqrt{x}\right) + \sqrt{x}\,\sqrt{1+x}$$

Result(type 3, 44 leaves):

$$\frac{\sqrt{\frac{x}{1+x}}\,\left(1+x\right)\left(2\sqrt{x^2+x} - \ln\left(\frac{1}{2} + x + \sqrt{x^2+x}\right)\right)}{2\sqrt{\left(1+x\right) x}}$$

Problem 253: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{1 - x^2 + \sqrt{5} + x^2 \sqrt{5}} \, dx$$

Optimal(type 3, 12 leaves, 2 steps):

$$\frac{\arctan\left(x\left(\frac{\sqrt{5}}{2} - \frac{1}{2}\right)\right)}{2}$$

Result(type 3, 31 leaves):

$$\frac{4 \arctan\left(\frac{4 x}{2 + 2\sqrt{5}}\right)}{\left(\sqrt{5} - 1\right)\left(2 + 2\sqrt{5}\right)}$$

Problem 257: Unable to integrate problem.

$$\int \sqrt{1 - x^2 + x\sqrt{x^2-1}} \, dx$$

Optimal(type 3, 49 leaves, ? steps):

$$\frac{3 \arcsin\left(x - \sqrt{x^2-1}\right)\sqrt{2}}{8} + \frac{\left(3 x + \sqrt{x^2-1}\right)\sqrt{1 - x^2 + x\sqrt{x^2-1}}}{4}$$

Result(type 8, 20 leaves):

$$\int \sqrt{1-x^2+x\sqrt{x^2-1}}\,dx$$

Problem 258: Unable to integrate problem.

$$\int \frac{\sqrt{-x+\sqrt{x}\,\sqrt{1+x}}}{\sqrt{1+x}}\,dx$$

Optimal(type 3, 46 leaves, ? steps):

$$-\frac{3\arcsin\left(\sqrt{x}-\sqrt{1+x}\right)\sqrt{2}}{4}+\frac{\left(\sqrt{x}+3\sqrt{1+x}\right)\sqrt{-x+\sqrt{x}\,\sqrt{1+x}}}{2}$$

Result(type 8, 23 leaves):

$$\int \frac{\sqrt{-x+\sqrt{x}\,\sqrt{1+x}}}{\sqrt{1+x}}\,dx$$

Problem 259: Result more than twice size of optimal antiderivative.

$$\int \frac{-x-2\sqrt{x^2+1}}{x+x^3+\sqrt{x^2+1}}\,dx$$

Optimal(type 3, 58 leaves, ? steps):

$$\operatorname{arctanh}\left(\left(x+\sqrt{x^2+1}\right)\sqrt{2+\sqrt{5}}\right)\sqrt{-2+2\sqrt{5}}-\arctan\left(\left(x+\sqrt{x^2+1}\right)\sqrt{-2+\sqrt{5}}\right)\sqrt{2+2\sqrt{5}}$$

Result(type 3, 437 leaves):

$$-\frac{\sqrt{5}\arctan\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{\sqrt{2+2\sqrt{5}}}-\frac{\arctan\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{\sqrt{2+2\sqrt{5}}}-\frac{\sqrt{5}\operatorname{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{\sqrt{-2+2\sqrt{5}}}+\frac{\operatorname{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{\sqrt{-2+2\sqrt{5}}}-\frac{\sqrt{x^2+1}}{2}-\frac{x}{2}$$
$$+\frac{3\sqrt{5}\operatorname{arctanh}\left(\frac{\sqrt{x^2+1}-x}{\sqrt{-2+\sqrt{5}}}\right)}{10\sqrt{-2+\sqrt{5}}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{x^2+1}-x}{\sqrt{-2+\sqrt{5}}}\right)}{2\sqrt{-2+\sqrt{5}}}+\frac{3\sqrt{5}\arctan\left(\frac{\sqrt{x^2+1}-x}{\sqrt{2+\sqrt{5}}}\right)}{10\sqrt{2+\sqrt{5}}}+\frac{\arctan\left(\frac{\sqrt{x^2+1}-x}{\sqrt{2+\sqrt{5}}}\right)}{2\sqrt{2+\sqrt{5}}}+\frac{1}{2\left(\sqrt{x^2+1}-x\right)}$$
$$+\frac{\operatorname{arctanh}\left(\frac{\sqrt{x^2+1}-x}{\sqrt{2+\sqrt{5}}}\right)}{2\sqrt{2+\sqrt{5}}}+\frac{\sqrt{5}\operatorname{arctanh}\left(\frac{\sqrt{x^2+1}-x}{\sqrt{2+\sqrt{5}}}\right)}{2\sqrt{2+\sqrt{5}}}-\frac{\arctan\left(\frac{\sqrt{x^2+1}-x}{\sqrt{-2+\sqrt{5}}}\right)}{2\sqrt{-2+\sqrt{5}}}+\frac{\sqrt{5}\arctan\left(\frac{\sqrt{x^2+1}-x}{\sqrt{-2+\sqrt{5}}}\right)}{2\sqrt{-2+\sqrt{5}}}$$

$$+\frac{2\sqrt{-2+\sqrt{5}}\,\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{x^2+1}-x}{\sqrt{-2+\sqrt{5}}}\right)}{5}-\frac{2\sqrt{5}\,\sqrt{2+\sqrt{5}}\,\arctan\left(\frac{\sqrt{x^2+1}-x}{\sqrt{2+\sqrt{5}}}\right)}{5}$$

Problem 260: Result more than twice size of optimal antiderivative.

$$\int \frac{x}{\sqrt{b\,d^4x^4+4\,b\,c\,d^3x^3+6\,b\,c^2d^2x^2+4\,b\,c^3d\,x+b\,c^4+a}}\,dx$$

Optimal(type 4, 195 leaves, 7 steps):

$$\frac{\operatorname{arctanh}\left(\frac{d^2\left(\frac{c}{d}+x\right)^2\sqrt{b}}{\sqrt{a+b\,d^4\left(\frac{c}{d}+x\right)^4}}\right)}{2\,d^2\sqrt{b}}$$

$$-\frac{1}{2\cos\left(2\arctan\left(\frac{b^{1/4}(d\,x+c)}{a^{1/4}}\right)\right)a^{1/4}b^{1/4}d^2\sqrt{a+b\,d^4\left(\frac{c}{d}+x\right)^4}}\left(c\sqrt{\cos\left(2\arctan\left(\frac{b^{1/4}(d\,x+c)}{a^{1/4}}\right)\right)^2}\operatorname{EllipticF}\left(\sin\left(2\right.\right.\right.$$

$$\left.\left.\left.\arctan\left(1\Big/a^{1/4}\left(b^{1/4}(d\,x+c)\right)\right)\right),\frac{\sqrt{2}}{2}\right)\left(\sqrt{a}+d^2\left(\frac{c}{d}+x\right)^2\sqrt{b}\right)\sqrt{\frac{a+b\,d^4\left(\frac{c}{d}+x\right)^4}{\left(\sqrt{a}+d^2\left(\frac{c}{d}+x\right)^2\sqrt{b}\right)^2}}\right)$$

Result(type 4, 1527 leaves):

$$\left(2\left(\frac{\frac{(-a\,b^3)^{1/4}}{b}-c}{d}-\frac{-\frac{I\,(-a\,b^3)^{1/4}}{b}-c}{d}\right)\sqrt{\frac{\left(\frac{-\frac{I\,(-a\,b^3)^{1/4}}{b}-c}{d}-\frac{\frac{I\,(-a\,b^3)^{1/4}}{b}-c}{d}\right)\left(x-\frac{\frac{(-a\,b^3)^{1/4}}{b}-c}{d}\right)}{\left(\frac{-\frac{I\,(-a\,b^3)^{1/4}}{b}-c}{d}-\frac{\frac{(-a\,b^3)^{1/4}}{b}-c}{d}\right)\left(x-\frac{\frac{I\,(-a\,b^3)^{1/4}}{b}-c}{d}\right)}}\left(x\right.\right.$$

$$- \frac{\frac{I\,(-a\,b^{3})^{1/4}}{b} - c}{d}\Bigg)$$

$$2\,\sqrt{\frac{\left(\frac{\frac{I\,(-a\,b^{3})^{1/4}}{b} - c}{d} - \frac{\frac{(-a\,b^{3})^{1/4}}{b} - c}{d}\right)\left(x - \frac{-\frac{(-a\,b^{3})^{1/4}}{b} - c}{d}\right)}{\left(\frac{-\frac{(-a\,b^{3})^{1/4}}{b} - c}{d} - \frac{\frac{(-a\,b^{3})^{1/4}}{b} - c}{d}\right)\left(x - \frac{\frac{I\,(-a\,b^{3})^{1/4}}{b} - c}{d}\right)}}$$

$$\sqrt{\frac{\left(\frac{\frac{I\,(-a\,b^{3})^{1/4}}{b} - c}{d} - \frac{\frac{(-a\,b^{3})^{1/4}}{b} - c}{d}\right)\left(x - \frac{-\frac{I\,(-a\,b^{3})^{1/4}}{b} - c}{d}\right)}{\left(\frac{-\frac{I\,(-a\,b^{3})^{1/4}}{b} - c}{d} - \frac{\frac{(-a\,b^{3})^{1/4}}{b} - c}{d}\right)\left(x - \frac{\frac{I\,(-a\,b^{3})^{1/4}}{b} - c}{d}\right)}}\;\Bigg(\frac{1}{d}\Bigg(\Bigg(\frac{I\,(-a\,b^{3})^{1/4}}{b}$$

$$- c\Bigg)\,\mathrm{EllipticF}\Bigg(\sqrt{\frac{\left(\frac{-\frac{I\,(-a\,b^{3})^{1/4}}{b} - c}{d} - \frac{\frac{I\,(-a\,b^{3})^{1/4}}{b} - c}{d}\right)\left(x - \frac{\frac{(-a\,b^{3})^{1/4}}{b} - c}{d}\right)}{\left(\frac{-\frac{I\,(-a\,b^{3})^{1/4}}{b} - c}{d} - \frac{\frac{(-a\,b^{3})^{1/4}}{b} - c}{d}\right)\left(x - \frac{\frac{I\,(-a\,b^{3})^{1/4}}{b} - c}{d}\right)}},$$

$$\sqrt{\frac{\left(\frac{\frac{I\,(-a\,b^{3})^{1/4}}{b} - c}{d} - \frac{-\frac{(-a\,b^{3})^{1/4}}{b} - c}{d}\right)\left(\frac{\frac{(-a\,b^{3})^{1/4}}{b} - c}{d} - \frac{-\frac{I\,(-a\,b^{3})^{1/4}}{b} - c}{d}\right)}{\left(\frac{\frac{(-a\,b^{3})^{1/4}}{b} - c}{d} - \frac{-\frac{(-a\,b^{3})^{1/4}}{b} - c}{d}\right)\left(\frac{\frac{I\,(-a\,b^{3})^{1/4}}{b} - c}{d} - \frac{-\frac{I\,(-a\,b^{3})^{1/4}}{b} - c}{d}\right)}}\Bigg)\Bigg) + \Bigg(\frac{\frac{(-a\,b^{3})^{1/4}}{b} - c}{d}$$

$$-\left(\frac{\frac{\mathrm{I}\left(-a\,b^{3}\right)^{1/4}}{b}-c}{d}\right)\mathrm{EllipticPi}\left(\sqrt{\frac{\left(\frac{-\frac{\mathrm{I}\left(-a\,b^{3}\right)^{1/4}}{b}-c}{d}-\frac{\frac{\mathrm{I}\left(-a\,b^{3}\right)^{1/4}}{b}-c}{d}\right)\left(x-\frac{\frac{\left(-a\,b^{3}\right)^{1/4}}{b}-c}{d}\right)}{\left(\frac{-\frac{\mathrm{I}\left(-a\,b^{3}\right)^{1/4}}{b}-c}{d}-\frac{\frac{\left(-a\,b^{3}\right)^{1/4}}{b}-c}{d}\right)\left(x-\frac{\frac{\mathrm{I}\left(-a\,b^{3}\right)^{1/4}}{b}-c}{d}\right)}},\right.$$

$$\left.\frac{\frac{-\frac{\mathrm{I}\left(-a\,b^{3}\right)^{1/4}}{b}-c}{d}-\frac{\frac{\left(-a\,b^{3}\right)^{1/4}}{b}-c}{d}}{\frac{-\frac{\mathrm{I}\left(-a\,b^{3}\right)^{1/4}}{b}-c}{d}-\frac{\frac{\mathrm{I}\left(-a\,b^{3}\right)^{1/4}}{b}-c}{d}},\sqrt{\frac{\left(\frac{\frac{\mathrm{I}\left(-a\,b^{3}\right)^{1/4}}{b}-c}{d}-\frac{-\frac{\left(-a\,b^{3}\right)^{1/4}}{b}-c}{d}\right)\left(\frac{\frac{\left(-a\,b^{3}\right)^{1/4}}{b}-c}{d}-\frac{-\frac{\mathrm{I}\left(-a\,b^{3}\right)^{1/4}}{b}-c}{d}\right)}{\left(\frac{\frac{\left(-a\,b^{3}\right)^{1/4}}{b}-c}{d}-\frac{-\frac{\left(-a\,b^{3}\right)^{1/4}}{b}-c}{d}\right)\left(\frac{\frac{\mathrm{I}\left(-a\,b^{3}\right)^{1/4}}{b}-c}{d}-\frac{-\frac{\mathrm{I}\left(-a\,b^{3}\right)^{1/4}}{b}-c}{d}\right)}}\right)\right)\right)$$

$$\Bigg/\left(\left(\frac{-\frac{\mathrm{I}\left(-a\,b^{3}\right)^{1/4}}{b}-c}{d}-\frac{\frac{\mathrm{I}\left(-a\,b^{3}\right)^{1/4}}{b}-c}{d}\right)\left(\frac{\frac{\mathrm{I}\left(-a\,b^{3}\right)^{1/4}}{b}-c}{d}\right.\right.$$

$$\left.\left.-\frac{\frac{\left(-a\,b^{3}\right)^{1/4}}{b}-c}{d}\right)\sqrt{b\,d^{4}\left(x-\frac{\frac{\left(-a\,b^{3}\right)^{1/4}}{b}-c}{d}\right)\left(x-\frac{\frac{\mathrm{I}\left(-a\,b^{3}\right)^{1/4}}{b}-c}{d}\right)\left(x-\frac{-\frac{\left(-a\,b^{3}\right)^{1/4}}{b}-c}{d}\right)\left(x-\frac{-\frac{\mathrm{I}\left(-a\,b^{3}\right)^{1/4}}{b}-c}{d}\right)}\right)$$

Problem 261: Result is not expressed in closed-form.

$$\int\frac{-c\,x^{4}+a}{\left(c\,d\,x^{4}+a\,e\,x^{2}+a\,d\right)\sqrt{c\,x^{4}+b\,x^{2}+a}}\,dx$$

Optimal(type 3, 44 leaves, 2 steps):

$$\frac{\operatorname{arctanh}\left(\frac{x\sqrt{-a\,e+b\,d}}{\sqrt{d}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}\right)}{\sqrt{d}\,\sqrt{-a\,e+b\,d}}$$

Result(type 7, 513 leaves):

$$-\frac{\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)}{4d\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{cx^4+bx^2+a}}$$

$$-\frac{1}{4d}\left(a\left(\sum_{\_\alpha=RootOf(cd\_Z^4+ae\_Z^2+ad)}\frac{1}{\_\alpha\left(2\_\alpha^2cd+ae\right)}\left((-\_\alpha^2e-2d)\left(-\frac{\operatorname{arctanh}\left(\frac{2\_\alpha^2cx^2+b\_\alpha^2+bx^2+2a}{2\sqrt{\frac{\_\alpha^2(-ae+bd)}{d}}\sqrt{cx^4+bx^2+a}}\right)}{\sqrt{\frac{\_\alpha^2(-ae+bd)}{d}}}\right.\right.\right.\right.$$

$$+\frac{1}{ad\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{cx^4+bx^2+a}}\Bigg(\sqrt{2}\,\_\alpha\left(\_\alpha^2cd\right.$$

$$\left.+ae\right)\sqrt{2+\frac{bx^2}{a}-\frac{x^2\sqrt{-4ac+b^2}}{a}}\,\sqrt{2+\frac{bx^2}{a}+\frac{x^2\sqrt{-4ac+b^2}}{a}}\,\mathrm{EllipticPi}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\right.$$

$$\left.\left.\left.\left.\left.\left.\frac{\_\alpha^2\sqrt{-4ac+b^2}\,cd+\_\alpha^2bcd+\sqrt{-4ac+b^2}\,ae+abe}{2adc},\frac{\sqrt{-\frac{b+\sqrt{-4ac+b^2}}{2a}}\,\sqrt{2}}{\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}\right)\right)\right)\right)\right)\right)$$

Problem 262: Unable to integrate problem.

$$\int\sqrt{\frac{x^n}{1+x^n}}\,dx$$

Optimal(type 5, 32 leaves, 3 steps):

$$\frac{2x\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{1}{2}+\frac{1}{n}\right],\left[\frac{3}{2}+\frac{1}{n}\right],-x^n\right)\sqrt{x^n}}{n+2}$$

Result(type 8, 15 leaves):

$$\int \sqrt{\frac{x^n}{1+x^n}}\, dx$$

Problem 263: Unable to integrate problem.

$$\int \frac{\sqrt{-a x^2 + b x \sqrt{\frac{a}{b^2} + \frac{a^2 x^2}{b^2}}}}{x \sqrt{\frac{a}{b^2} + \frac{a^2 x^2}{b^2}}}\, dx$$

Optimal(type 3, 38 leaves, 2 steps):

$$\frac{b \arcsin\left(\frac{a x - b \sqrt{\frac{a}{b^2} + \frac{a^2 x^2}{b^2}}}{\sqrt{a}}\right) \sqrt{2}}{\sqrt{a}}$$

Result(type 8, 54 leaves):

$$\int \frac{\sqrt{-a x^2 + b x \sqrt{\frac{a}{b^2} + \frac{a^2 x^2}{b^2}}}}{x \sqrt{\frac{a}{b^2} + \frac{a^2 x^2}{b^2}}}\, dx$$

Problem 264: Unable to integrate problem.

$$\int \frac{\sqrt{x \left(-a x + b \sqrt{\frac{a}{b^2} + \frac{a^2 x^2}{b^2}}\right)}}{x \sqrt{\frac{a}{b^2} + \frac{a^2 x^2}{b^2}}}\, dx$$

Optimal(type 3, 38 leaves, 3 steps):

$$\frac{b \arcsin\left(\frac{a x - b \sqrt{\frac{a}{b^2} + \frac{a^2 x^2}{b^2}}}{\sqrt{a}}\right) \sqrt{2}}{\sqrt{a}}$$

Result(type 8, 53 leaves):

$$\int \frac{\sqrt{x\left(-a x+b\sqrt{\frac{a}{b^{2}}+\frac{a^{2} x^{2}}{b^{2}}}\right)}}{x\sqrt{\frac{a}{b^{2}}+\frac{a^{2} x^{2}}{b^{2}}}}\,dx$$

Problem 265: Result more than twice size of optimal antiderivative.

$$\int \frac{-\sqrt{x-4}+x\sqrt{x-4}-4\sqrt{-1+x}+x\sqrt{-1+x}}{\left(x^{2}-5 x+4\right)\left(1+\sqrt{x-4}+\sqrt{-1+x}\right)}\,dx$$

Optimal(type 3, 15 leaves, 3 steps):

$$2 \ln\left(1+\sqrt{x-4}+\sqrt{-1+x}\right)$$

Result(type 3, 146 leaves):

$$\frac{\ln(-5+x)}{2}-\frac{\ln\left(1+\sqrt{x-4}\right)}{2}+\frac{\ln\left(-1+\sqrt{x-4}\right)}{2}+\frac{\ln\left(\sqrt{-1+x}+2\right)}{2}-\frac{\ln\left(\sqrt{-1+x}-2\right)}{2}+\frac{7\sqrt{x-4}\,\sqrt{-1+x}\,\operatorname{arctanh}\left(\frac{-17+5 x}{4\sqrt{x^{2}-5 x+4}}\right)}{4\sqrt{x^{2}-5 x+4}}$$
$$+\frac{\sqrt{x-4}\,\sqrt{-1+x}\left(2 \ln\left(-\frac{5}{2}+x+\sqrt{x^{2}-5 x+4}\right)-5 \operatorname{arctanh}\left(\frac{-17+5 x}{4\sqrt{x^{2}-5 x+4}}\right)\right)}{4\sqrt{x^{2}-5 x+4}}$$

Problem 266: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{x}{\sqrt{5308416 c^{4} x^{4}+576000 b^{2} c^{2} x^{2}+576000 b^{3} c x-44375 b^{4}}}\,dx$$

Optimal(type 3, 173 leaves, 1 step):

$$\frac{1}{18432 c^{2}}\Big(\ln\Big(20738073600000000 b^{8} c^{4}+597005697024000000 b^{6} c^{6} x^{2}+2583100705996800000 b^{5} c^{7} x^{3}+9510507144806400000 b^{4} c^{8} x^{4}$$
$$+21641687369515008000 b^{3} c^{9} x^{5}+32462531054272512000 b^{2} c^{10} x^{6}+14958734309808773 5296 c^{12} x^{8}+5308416\left(12230590464 c^{10} x^{6}\right.$$
$$\left.+1990656000 b^{2} c^{8} x^{4}+1105920000 b^{3} c^{7} x^{3}+38880000 b^{4} c^{6} x^{2}+79200000 b^{5} c^{5} x+12203125 b^{6} c^{4}\right)$$
$$\sqrt{5308416 c^{4} x^{4}+576000 b^{2} c^{2} x^{2}+576000 b^{3} c x-44375 b^{4}}\Big)\Big)$$

Result(type 4, 1596 leaves):

$$\Bigg(\left(\frac{5\, RootOf\left(\_Z^4 + 10\, \_Z^2 + 96\, \_Z - 71, index = 1 \right) b}{48\, c} \right.$$

$$\left. - \frac{5\, RootOf\left(\_Z^4 + 10\, \_Z^2 + 96\, \_Z - 71, index = 4 \right) b}{48\, c} \right)$$

$$\left(\left(\left(\frac{5\, RootOf\left(\_Z^4 + 10\, \_Z^2 + 96\, \_Z - 71, index = 4 \right) b}{48\, c} - \frac{5\, RootOf\left(\_Z^4 + 10\, \_Z^2 + 96\, \_Z - 71, index = 2 \right) b}{48\, c} \right) \left(x - \frac{5\, RootOf\left(\_Z^4 + 10\, \_Z^2 + 96\, \_Z - 71, index = \right.}{48\, c} \right. \right. \right.$$

$$\left. \left. \left. - \frac{5\, RootOf\left(\_Z^4 + 10\, \_Z^2 + 96\, \_Z - 71, index = 1 \right) b}{48\, c} \right) \left(x - \frac{5\, RootOf\left(\_Z^4 + 10\, \_Z^2 + 96\, \_Z - 71, index = 2 \right) b}{48\, c} \right) \right) \right)^{1/2} \Bigg(x$$

$$\left. - \frac{5\, RootOf\left(\_Z^4 + 10\, \_Z^2 + 96\, \_Z - 71, index = 2 \right) b}{48\, c} \right)$$

2

$$\Bigg(\bigg(\Big(\frac{5\,RootOf(\_Z^4+10\,\_Z^2+96\,\_Z-71, index=2)\,b}{48\,c}-\frac{5\,RootOf(\_Z^4+10\,\_Z^2+96\,\_Z-71, index=1)\,b}{48\,c}\Big)\Big(x$$

$$-\frac{5\,RootOf(\_Z^4+10\,\_Z^2+96\,\_Z-71, index=3)\,b}{48\,c}\Big)\bigg)\bigg/\bigg(\Big(\frac{5\,RootOf(\_Z^4+10\,\_Z^2+96\,\_Z-71, index=3)\,b}{48\,c}$$

$$-\frac{5\,RootOf(\_Z^4+10\,\_Z^2+96\,\_Z-71, index=1)\,b}{48\,c}\Big)\Big(x-\frac{5\,RootOf(\_Z^4+10\,\_Z^2+96\,\_Z-71, index=2)\,b}{48\,c}\Big)\bigg)\Bigg)$$

1/2

$$\Bigg(\bigg(\Big(\frac{5\,RootOf(\_Z^4+10\,\_Z^2+96\,\_Z-71, index=2)\,b}{48\,c}-\frac{5\,RootOf(\_Z^4+10\,\_Z^2+96\,\_Z-71, index=1)\,b}{48\,c}\Big)\Big(x$$

$$-\frac{5\,RootOf(\_Z^4+10\,\_Z^2+96\,\_Z-71, index=4)\,b}{48\,c}\Big)\bigg)\bigg/\bigg(\Big(\frac{5\,RootOf(\_Z^4+10\,\_Z^2+96\,\_Z-71, index=4)\,b}{48\,c}$$

$$- \frac{5\, RootOf(\_Z^4 + 10\,\_Z^2 + 96\,\_Z - 71, index = 1)\, b}{48\, c}\Bigg)\Bigg(x - \frac{5\, RootOf(\_Z^4 + 10\,\_Z^2 + 96\,\_Z - 71, index = 2)\, b}{48\, c}\Bigg)\Bigg)\Bigg)^{1/2} \Bigg(\frac{1}{48\, c}\Bigg(5\, RootOf(\_Z^4$$

$$+ 10\,\_Z^2 + 96\,\_Z - 71, index = 2)$$

$$b$$

$$\mathrm{EllipticF}\Bigg(\Bigg(\Bigg(\Bigg(\frac{5\, RootOf(\_Z^4 + 10\,\_Z^2 + 96\,\_Z - 71, index = 4)\, b}{48\, c} - \frac{5\, RootOf(\_Z^4 + 10\,\_Z^2 + 96\,\_Z - 71, index = 2)\, b}{48\, c}\Bigg)\Bigg(x$$

$$- \frac{5\, RootOf(\_Z^4 + 10\,\_Z^2 + 96\,\_Z - 71, index = 1)\, b}{48\, c}\Bigg)\Bigg) \Bigg/ \Bigg(\Bigg(\frac{5\, RootOf(\_Z^4 + 10\,\_Z^2 + 96\,\_Z - 71, index = 4)\, b}{48\, c}$$

$$- \frac{5\, RootOf(\_Z^4 + 10\,\_Z^2 + 96\,\_Z - 71, index = 1)\, b}{48\, c}\Bigg)\Bigg(x - \frac{5\, RootOf(\_Z^4 + 10\,\_Z^2 + 96\,\_Z - 71, index = 2)\, b}{48\, c}\Bigg)\Bigg)\Bigg)^{1/2},$$

$$\Bigg(\Bigg(\Bigg(\frac{5\, RootOf(\_Z^4 + 10\,\_Z^2 + 96\,\_Z - 71, index = 2)\, b}{48\, c} - \frac{5\, RootOf(\_Z^4 + 10\,\_Z^2 + 96\,\_Z - 71, index = 3)\, b}{48\, c}\Bigg)\Bigg(\frac{5\, RootOf(\_Z^4 + 10\,\_Z^2 + 96\,\_Z - 71, index = 1)}{48\, c}$$

$$- \frac{5\, RootOf(\_Z^4 + 10\,\_Z^2 + 96\,\_Z - 71, index = 3)\, b}{48\, c}\Bigg)\Bigg(\frac{5\, RootOf(\_Z^4 + 10\,\_Z^2 + 96\,\_Z - 71, index = 2)\, b}{48\, c}$$

$$- \frac{5\, RootOf(\_Z^4 + 10\_Z^2 + 96\_Z - 71, index = 4)\, b}{48\, c} \Bigg) \Bigg) \Bigg) ^{1/2} \Bigg) \Bigg) + \Bigg(\frac{5\, RootOf(\_Z^4 + 10\_Z^2 + 96\_Z - 71, index = 1)\, b}{48\, c}$$

$$- \frac{5\, RootOf(\_Z^4 + 10\_Z^2 + 96\_Z - 71, index = 2)\, b}{48\, c} \Bigg)$$

$$\mathrm{EllipticPi} \Bigg(\Bigg(\Bigg(\Bigg(\frac{5\, RootOf(\_Z^4 + 10\_Z^2 + 96\_Z - 71, index = 4)\, b}{48\, c} - \frac{5\, RootOf(\_Z^4 + 10\_Z^2 + 96\_Z - 71, index = 2)\, b}{48\, c} \Bigg) \Bigg(x$$

$$- \frac{5\, RootOf(\_Z^4 + 10\_Z^2 + 96\_Z - 71, index = 1)\, b}{48\, c} \Bigg) \Bigg) \Bigg/ \Bigg(\Bigg(\frac{5\, RootOf(\_Z^4 + 10\_Z^2 + 96\_Z - 71, index = 4)\, b}{48\, c}$$

$$- \frac{5\, RootOf(\_Z^4 + 10\_Z^2 + 96\_Z - 71, index = 1)\, b}{48\, c} \Bigg) \Bigg(x - \frac{5\, RootOf(\_Z^4 + 10\_Z^2 + 96\_Z - 71, index = 2)\, b}{48\, c} \Bigg) \Bigg) \Bigg) ^{1/2},$$

$$\frac{\frac{5\, RootOf(\_Z^4 + 10\_Z^2 + 96\_Z - 71, index = 4)\, b}{48\, c} - \frac{5\, RootOf(\_Z^4 + 10\_Z^2 + 96\_Z - 71, index = 1)\, b}{48\, c}}{\frac{5\, RootOf(\_Z^4 + 10\_Z^2 + 96\_Z - 71, index = 4)\, b}{48\, c} - \frac{5\, RootOf(\_Z^4 + 10\_Z^2 + 96\_Z - 71, index = 2)\, b}{48\, c}},$$

$$\Bigg(\Bigg(\Bigg(\frac{5\, RootOf(\_Z^4 + 10\_Z^2 + 96\_Z - 71, index = 2)\, b}{48\, c} - \frac{5\, RootOf(\_Z^4 + 10\_Z^2 + 96\_Z - 71, index = 3)\, b}{48\, c} \Bigg) \Bigg(\frac{5\, RootOf(\_Z^4 + 10\_Z^2 + 96\_Z - 71, index = 1)\, b}{48\, c}$$

$$- \frac{5\, RootOf(\_Z^4 + 10\_Z^2 + 96\_Z - 71, index = 4)\, b}{48\, c} \Bigg) \Bigg) \Bigg) ^{1/2} \Bigg) \Bigg) \Bigg) \Bigg/ \Bigg(1152 \Bigg(\frac{5\, RootOf(\_Z^4 + 10\_Z^2 + 96\_Z - 71, index = 4)\, b}{48\, c}$$

$$- \frac{5\, RootOf(\_Z^4 + 10\_Z^2 + 96\_Z - 71, index = 2)\, b}{48\, c} \Bigg) \Bigg(\frac{5\, RootOf(\_Z^4 + 10\_Z^2 + 96\_Z - 71, index = 2)\, b}{48\, c}$$

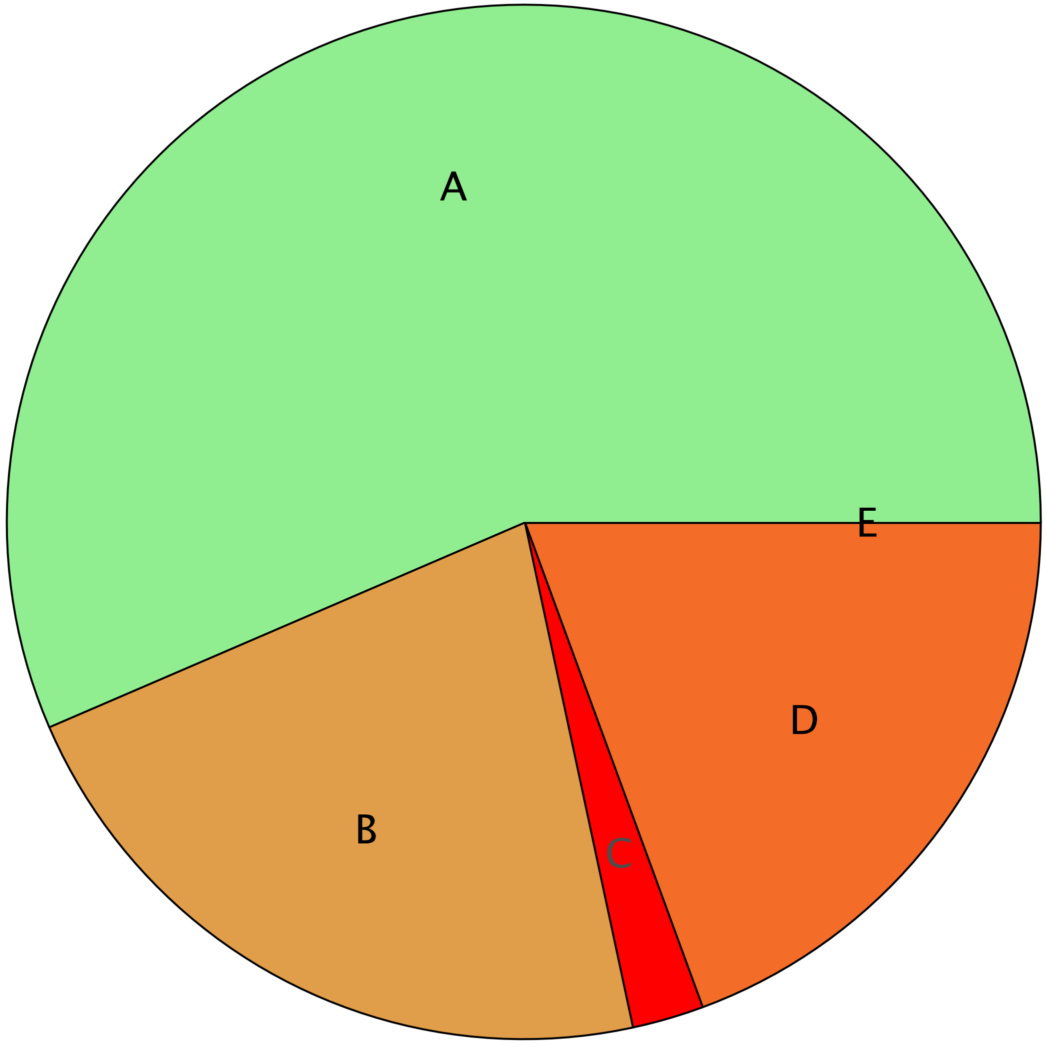
$$- \frac{5\, RootOf\left(\_Z^4 + 10\,\_Z^2 + 96\,\_Z - 71, index = 1\right) b}{48\, c}\Bigg)$$

$$\left(c^4 \left(x - \frac{5\, RootOf\left(\_Z^4 + 10\,\_Z^2 + 96\,\_Z - 71, index = 1\right) b}{48\, c}\right) \left(x - \frac{5\, RootOf\left(\_Z^4 + 10\,\_Z^2 + 96\,\_Z - 71, index = 2\right) b}{48\, c}\right) \left(x - \frac{5\, RootOf\left(\_Z^4 + 10\,\_Z^2 + 96\,\_Z - 7\right.}{48\, c}\right.\right.$$

$$\left.\left.\left. - \frac{5\, RootOf\left(\_Z^4 + 10\,\_Z^2 + 96\,\_Z - 71, index = 4\right) b}{48\, c}\right)\right)^{1/2}\right)$$

Summary of Integration Test Results

402 integration problems



A - 227 optimal antiderivatives
B - 88 more than twice size of optimal antiderivatives
C - 9 unnecessarily complex antiderivatives
D - 78 unable to integrate problems
E - 0 integration timeouts